Generalized Encoding and Decoding Functions for a Cylindrical Ambisonic Sound System

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***Abstract*—In this letter, we present generalized encoding and decoding functions for an ambisonic system. Our approach provides matrix representations for optimum encoding and decoding for both odd and even number of N loudspeakers. Particularly, we present ambisonic components (signals or channels) for even N and a parameterized decoding equation for the optimum localization.**

***Index Terms*—Ambisonic surround system, explicit encoding functions for even number of loudspeakers, generalized encoding and decoding functions for an ambisonic system, optimum localization for surround systems.**

I. INTRODUCTION

AMBISONIC SYSTEMS will be evaluated for their ability to reproduce a single sound source by means of a loudspeaker layout. Using a set of loudspeakers it is a key to determine their optimal feeding, which minimizes the difference between the reconstructed wave field and an ideal plane wave in the listening area [2], [3]. An accurate reproduction of sound field is dependent on the encoding functions. Recently, Poletti presented a design of the optimum encoding functions for an ambisonic system, where the angular sinc functions consisting of the circular sinc functions for the optimum localization was proposed as follows (see also [3]).

A family of angular sinc functions will be described that produce the optimum localization for surround systems of all orders M and N, where M and N denote ambisonic order and number of loudspeakers, respectively.

This letter is concerned with only sounds in a 360° horizontal plane. Assume that the plane wave is arriving at an angle ψ, and the listening position is at a radial distance r at an angle ϕ, both counterclockwise with respect to the x axis. Then, the original plane wave is expressed as

$$S_\psi = P_\psi e^{ikr\cos(\phi-\psi)} \tag{1.1}$$

where k is the wavenumber, and P_ψ is the pressure of the plane wave. The aim of the ambisonic system is to be able to reproduce this plane wave in the center of the listening area.

In [3], it was presented that the encoding functions that produce optimum matching of the first-order spherical harmonics of an ideal plane wave and the signal synthesized by the ambisonic system are

$$w_n(\psi) = \frac{1}{N}\left[1 + 2\sum_{m=1}^{M}\cos m(\psi-\phi_n)\right] \tag{1.2}$$

where $\phi_n = 2\pi n/N$ is the angle of the nth loudspeaker; M is the system-order; and the $1/N$ term is the constant of proportionality. The sinc function is defined as

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}. \tag{1.3}$$

The sinc function occurs in a number of applications such as reconstruction of a continuous signal from its uniformly sampled values. The circular sinc function is defined as

$$\text{csinc}_N(\psi) = \frac{\sin\left(\frac{N\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}. \tag{1.4}$$

For N odd this can be written as

$$\begin{aligned}\text{csinc}_N(\psi) &= \sum_{n=-(N-1)/2}^{(N-1)/2} e^{jn\theta}\\ &= 1 + 2\sum_{n=1}^{(N-1)/2}\cos(n\psi) + 2\sum_{n=1}^{(N-1)/2}\sin(n\psi).\end{aligned} \tag{1.5}$$

In order to preserve the periodicity of 360°, the even N function with respect to the criterion for optimality is expressed as

$$\begin{aligned}g(\psi) = 1 &+ 2\sum_{n=1}^{(N-2)/2}\cos(n\psi) + \cos\left(\frac{N\psi}{2}\right)\\ &+2\sum_{n=1}^{(N-2)/2}\sin(n\psi) + \sin\left(\frac{N\psi}{2}\right) \quad \text{for even } N.\end{aligned} \tag{1.6}$$

Hence, a set of angular sinc functions that have 360° periodicity for integers N can be defined as

$$\text{asinc}_N(\psi) = \begin{cases}\text{csinc}_N(\psi), & \text{for odd } N\\ g(\psi), & \text{for even } N.\end{cases} \tag{1.7}$$

It was mentioned in [3] that the asinc functions for odd N are equal to those derived in [5] and [1]. There are also well-defined encoding functions for even N. However, any explicit encoding functions for even N have not been shown clearly. To solve the odd–even number problem, Poletti proposed another encoding function; however, the associated encoding matrix-equation has no unique solution, since this encoding matrix

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is overdetermined [4]. Therefore, we present explicitly optimum encoding functions and corresponding decoding functions for both odd and even N. Especially, we will present a parameterized decoding equation for the optimum localization.

II. Main Results

Sound sources are assumed to be in a distance from the listener, namely in the far field, so that the wave fronts produced can be considered as a plane near the listening point. The reproduced plane wave is characterized by its incidence angle ψ according to the x axis and its pressure signal P_ψ at the center-point ($x = 0$, $y = 0$). Similarly, plane waves associated with the N loudspeakers are defined by angles ϕ_n and pressure signals P_n. Unitary vectors $\mathbf{u}_\psi = (\cos\psi, \sin\psi)$ and $\mathbf{u}_n = (\cos\phi_n, \sin\phi_n)$ are also used (see [1] and [2]).

Pressure signals measured at point $\mathbf{r} = r \cdot \mathbf{u}_\theta$ (with $\mathbf{u}_\theta = (\cos\theta, \sin\theta)$) are then defined in frequency domain by

$$S_\psi = P_\psi e^{ik\cdot\mathbf{u}_\psi\cdot\mathbf{u}} = P_\psi e^{ikr\cos(\theta-\psi)} \tag{2.8}$$
$$S_n = P_n e^{ik\cdot\mathbf{u}_n\cdot\mathbf{r}} = P_n e^{ikr\cos(\theta-\phi_n)} \tag{2.9}$$

where $k = 2\pi f/c$ is the wavenumber associated with the frequency f. Considering horizontal encoding of a single source (P_ψ, ψ), ambisonic components of order M and for odd N are defined by

$$\begin{aligned} W &= \alpha_0 P_\psi, \\ X_1 &= \alpha_1 P_\psi \cos\psi \\ Y_1 &= \alpha_1 P_\psi \sin\psi \\ X_2 &= \alpha_2 P_\psi \cos 2\psi \\ Y_2 &= \alpha_2 P_\psi \sin 2\psi \\ &\vdots \\ X_M &= \alpha_M P_\psi \cos M\psi \\ Y_M &= \alpha_M P_\psi \sin M\psi. \end{aligned} \tag{2.10}$$

For convenience, let $\alpha_0 = 1$, $\alpha_i = \sqrt{2}$, $i = 1..M$, and using matrix representation for encoding and decoding functions, we express the encoding matrix for the second-order ambisonic system

$$\begin{bmatrix} W \\ X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{bmatrix} = M_e \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_N \end{bmatrix}$$
$$M_e = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \sqrt{2}\cos\phi_1 & \sqrt{2}\cos\phi_2 & \cdots & \sqrt{2}\cos\phi_N \\ \sqrt{2}\sin\phi_1 & \sqrt{2}\sin\phi_2 & \cdots & \sqrt{2}\sin\phi_N \\ \sqrt{2}\cos 2\phi_1 & \sqrt{2}\cos 2\phi_2 & \cdots & \sqrt{2}\cos 2\phi_N \\ \sqrt{2}\sin 2\phi_1 & \sqrt{2}\sin 2\phi_2 & \cdots & \sqrt{2}\sin 2\phi_N \end{bmatrix}. \tag{2.11}$$

The encoding matrix for the second-order ambisonic system can be expressed as

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_N \end{bmatrix} = M_d \cdot \begin{bmatrix} W \\ X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{bmatrix} \quad M_d = M_e^T \cdot \left(M_e \cdot M_e^T\right)^{-1} \tag{2.12}$$

if the encoding matrix M_e has full row rank. Note that $M_d = (1/N)M_e^T$ if the loudspeaker layout is a regular polygon. Poletti proposed the following encoding functions.

For example, 1) for four loudspeakers with second-order and five channels and 2) for six loudspeakers with third-order and seven channels, the associated encoding functions asinc_N are presented, respectively, as follows:

$$1 + 2\cos(\psi) + \cos(2\psi) + 2\sin(\psi) + \sin(2\psi) \tag{2.13}$$

and

$$\begin{aligned} &1 + 2\cos(\psi) + 2\cos(2\psi) + \cos(3\psi) \\ &\qquad + 2\sin(\psi) + 2\sin(2\psi) + \sin(3\psi). \end{aligned} \tag{2.14}$$

From (2.12), we infer that the right-inverse of the matrix M_e does not exist, since the matrix M_e does not have full row rank. Therefore, a generalized inverse of M_e is not unique [4].

Now, we will present explicitly encoding functions and corresponding decoding functions for both odd and even N. From the angular sinc function (1.7), we will derive an encoding function as follows (e.g., $N = 6$):

$$\begin{aligned} \mathrm{asinc}_N(\psi) = &1 + 2\cos(\psi) + 2\cos(2\psi) + \cos(3\psi) \\ &+ 2\sin(\psi) + 2\sin(2\psi) + \sin(3\psi). \end{aligned}$$

The corresponding encoding function is expressed as (2.15), shown at at the top of the next page. Then, the associated decoding function is

$$\begin{aligned} P_n &= \frac{1}{6}\left[1 \quad \sqrt{2}\cos\phi_n \quad \sqrt{2}\sin\phi_n \quad \sqrt{2}\cos 2\phi_n \quad \sqrt{2}\sin 2\phi_n\right] \\ &\quad \cdot \begin{bmatrix} P_\psi \\ P_\psi\sqrt{2}\cos\psi \\ P_\psi\sqrt{2}\sin\psi \\ P_\psi\sqrt{2}\left(\cos 2\psi + \frac{1}{2}\cos 3\psi\right) \\ P_\psi\sqrt{2}\left(\sin 2\psi + \frac{1}{2}\sin 3\psi\right) \end{bmatrix} \\ &= M_d(n) \cdot \begin{bmatrix} W \\ X_1 \\ Y_1 \\ X_2 + \frac{1}{2}X_3 \\ Y_2 + \frac{1}{2}Y_3 \end{bmatrix} \triangleq M_d(n) \cdot \begin{bmatrix} W \\ X_1 \\ Y_1 \\ \hat{X}_2 \\ \hat{Y}_2 \end{bmatrix} \end{aligned} \tag{2.16}$$

where $M_d(n)$ denotes the nth row of M_d.

Therefore, the decoding equation yields:

$$\begin{aligned} P_n = \frac{1}{6}\Big[&P_\psi + \sqrt{2}\cos\phi_n X_1 + \sqrt{2}\sin\phi_n Y_1 \\ &+ \sqrt{2}\cos 2\phi_n \hat{X}_2 + \sqrt{2}\sin 2\phi_n \hat{Y}_2\Big]. \end{aligned} \tag{2.17}$$

Analogously, we obtain the encoding and decoding functions of higher orders. Note that $w_n(\psi)$ can be expressed by the product

$$
\begin{aligned}
w_n(\psi) &= \frac{1}{6}\left[1 + 2\cos(2\psi - 2\phi_n) + \cos(3\psi - 2\phi_n)\right] \\
&= \frac{1}{6}\Bigg[1 + 2\cos\psi \cdot \cos\phi_n 2\left(\cos 2\psi + \frac{1}{2}\cos 3\psi\right)\cos 2\phi_n \\
&\qquad + 2\sin\psi \cdot \sin\phi_n + 2\left(\sin 2\psi + \frac{1}{2}\sin 3\psi\right)\sin 2\phi_n\Bigg] \\
&= \frac{1}{6}\begin{bmatrix} 1 & \sqrt{2}\cos\psi & \sqrt{2}\sin\psi & \sqrt{2}\left(\cos 2\psi + \frac{1}{2}\cos 3\psi\right) & \sqrt{2}\left(\sin 2\psi + \frac{1}{2}\sin 3\psi\right)\end{bmatrix} \cdot \begin{bmatrix} 1 \\ \sqrt{2}\cos\phi_n \\ \sqrt{2}\sin\phi_n \\ \sqrt{2}\cos 2\phi_n \\ \sqrt{2}\sin 2\phi_n \end{bmatrix} \qquad (2.15)
\end{aligned}
$$

$$
w_n(\psi) = \begin{cases} \frac{1}{N}\left[1 + 2\sum_{m=1}^{M}\cos(m(\psi - \phi_n))\right], & \text{for odd } N \\ \frac{1}{N}\left[1 + 2\sum_{m=1}^{M}\cos(m(\psi - \phi_n)) + \cos((M+1)\psi - M\phi_n)\right], & \text{for even } N \end{cases} \qquad (2.18)
$$

$$
P_n = \begin{cases} \frac{1}{N}\left[\beta_0 P_\psi + \sum_{m=1}^{M}\beta_m(\cos\phi_n X_m + \sin\phi_n Y_m)\right], & \text{for odd } N \\ \frac{1}{N}\left[\beta_0 P_\psi + \sum_{m=1}^{M-1}\beta_m(\cos\phi_n X_m + \sin\phi_n Y_m) + \beta_M\left(\cos\phi_n \hat{X}_M + \sin\phi_n \hat{Y}_M\right)\right], & \text{for even } N \end{cases} \qquad (2.20)
$$

of the coefficients of $g(\psi)$ and the nth column of the encoding matrix. By virtue of the asinc functions, the general form of the optimal encoding function are proposed as follows.

For $n = 0, 1, 2, \ldots, N-1$, it holds (2.18), found at the top of the page. Hence, ambisonic components of order M for even N are defined by

$$
\begin{aligned}
W &= \alpha_0 P_\psi, \\
&\vdots \\
X_{M-1} &= \alpha_{M-1} P_\psi \cos(M-1)\psi \\
Y_{M-1} &= \alpha_{M-1} P_\psi \sin(M-1)\psi \\
\tilde{X}_M &= \alpha_M P_\psi\left(\cos M\psi + \frac{1}{2}\cos(M+1)\psi\right) \\
\tilde{Y}_M &= \alpha_M P_\psi\left(\sin M\psi + \frac{1}{2}\sin(M+1)\psi\right) \qquad (2.19)
\end{aligned}
$$

for $M \geq 1$ where X_0 and Y_0 are not defined.

A. Generalized Decoding Equation

The generalized decoding equation, for all N, holds (2.20), found at the top of the page. In order to ensure the optimum localization for surround systems of all orders M and N from the (2.10), (2.19), (2.15), (2.18), and (2.20), we infer that $\alpha_0 \cdot \beta_0 = 1$ and $\alpha_i \cdot \beta_i = 2$, $i \in 1 \ldots M$.

III. Conclusion

Using matrix representations, we have proposed generalized encoding and decoding functions for an ambisonic system. The ambisonic order (especially, for N even) is always reduced. However, the last term of the ambisonic components includes the component of the usual order (one order higher). For the optimum localization, the parameterization of the decoding equations are also proposed. Moreover, we have presented the ambisonic components for even N. Our approach is unique, whereas the approach presented in [3] is not unique. In general, these functions depend on an odd or even number of loudspeakers.

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