A Comparison of Wave Field Synthesis and Higher-Order Ambisonics with Respect to Physical Properties and Spatial Sampling

Sascha Spors and Jens Ahrens

Deutsche Telekom Laboratories
Quality and Usability Laboratory
Technische Universität Berlin

125th Convention of the AES
San Francisco, Oct. 3rd, 2008

Introduction

For high-resolution sound reproduction, based on a physical recreation of a desired wave field, two alternative approaches exist

- Wave Field Synthesis (WFS)
- Higher-order Ambisonics (HOA)

The commons and differences of both approaches in terms of their physical foundations and practical realization are not fully understood.

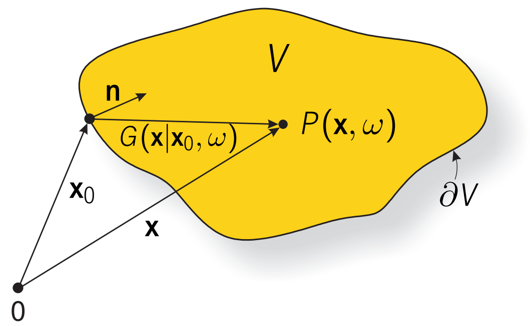
Here: comparison of both approaches under the following assumptions

- near-field compensated HOA without en-/decoding stage
- two-dimensional reproduction using a circular loudspeaker setup

Fundamentals of Sound Field Reproduction

The Kirchhoff-Helmholtz integral provides the solution of the homogeneous wave equation with respect to inhomogeneous boundary conditions

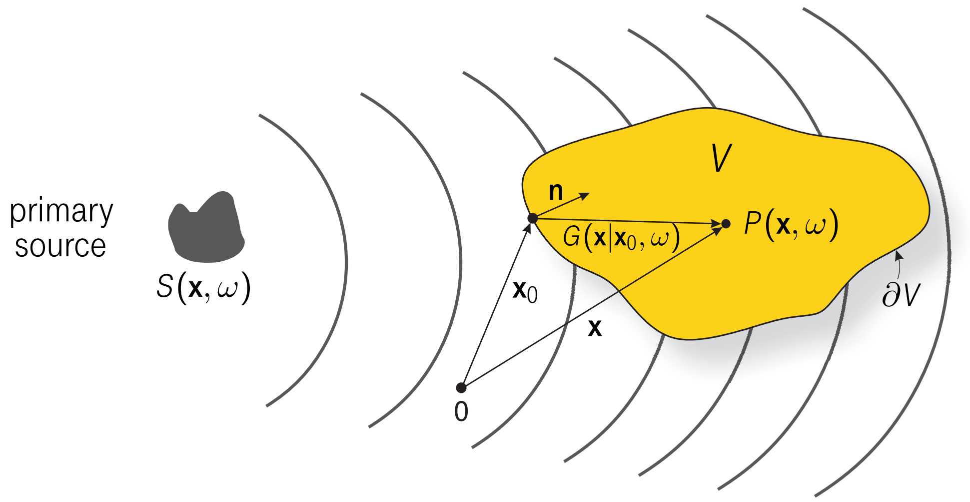
$$\boxed{P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_0,\omega) - P(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) dS_0}$$



Fundamentals of Sound Field Reproduction

The field of a primary source $S(\mathbf{x},\omega)$ within the area V is uniquely given by its pressure and pressure gradient on the boundary ∂V

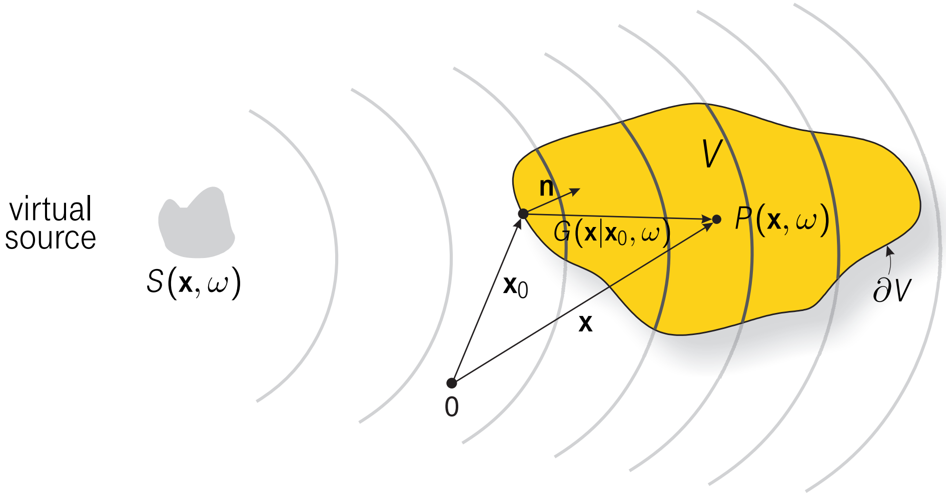
$$\boxed{P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) dS_0}$$



Fundamentals of Sound Field Reproduction

The Green's function and its gradient can be interpreted as (secondary) sources that generate the field of a virtual source $S(\mathbf{x},\omega)$ inside the listening area V

$$\boxed{P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) \, dS_0}$$



Fundamentals of Sound Field Reproduction

The theoretical basis of sound field reproduction is given by the Kirchhoff-Helmholtz integral

$$\boxed{P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) \, dS_0}$$

The explicit form of the Green's function depends on the dimensionality

- two-dimensional reproduction $\longrightarrow$ secondary line sources
- three-dimensional reproduction $\longrightarrow$ secondary point sources

The need for two types of secondary sources is postulated

- monopole sources
- dipole sources

Elimination of Dipole Secondary Sources

- halves the number of required secondary sources
- monopole sources can be realized quite well by loudspeakers

Different schemes to eliminate the dipole secondary sources

1. **Modification of Green's function used in the Kirchhoff-Helmholtz integral**
   - assumption of a Neumann Green's function
   - secondary sources may be hard to realize for complex geometries
   - basis of Wave Field Synthesis
2. **The 'Simple Source Approach'**
   - provides formulation for monopole only reproduction
   - driving function is given by considering a disjunct interior/exterior problem
   - basis of higher-order Ambisonics

Wave Field Synthesis

Elimination of secondary dipole sources by using a Neumann Green's function

$$\boxed{P(\mathbf{x},\omega) = -\oint_{\partial V} \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x_0},\omega)\; G_\mathrm{N}(\mathbf{x}|\mathbf{x}_0,\omega)\; dS_0}$$

where $G_\mathrm{N}(\mathbf{x}|\mathbf{x}_0,\omega)$ is a Neumann Green's function with $\frac{\partial}{\partial \mathbf{n}} G_\mathrm{N}(\mathbf{x}|\mathbf{x}_0,\omega)\Big|_{\mathbf{x}_0\in\partial V} = 0.$

Fundamentals of WFS

- application of Neumann Green's function for linear/planar secondary source contour, realization by secondary point sources
- sensible selection of active secondary sources for curved ∂V
- no exact reproduction for non-linear/planar systems

Higher-Order Ambisonics

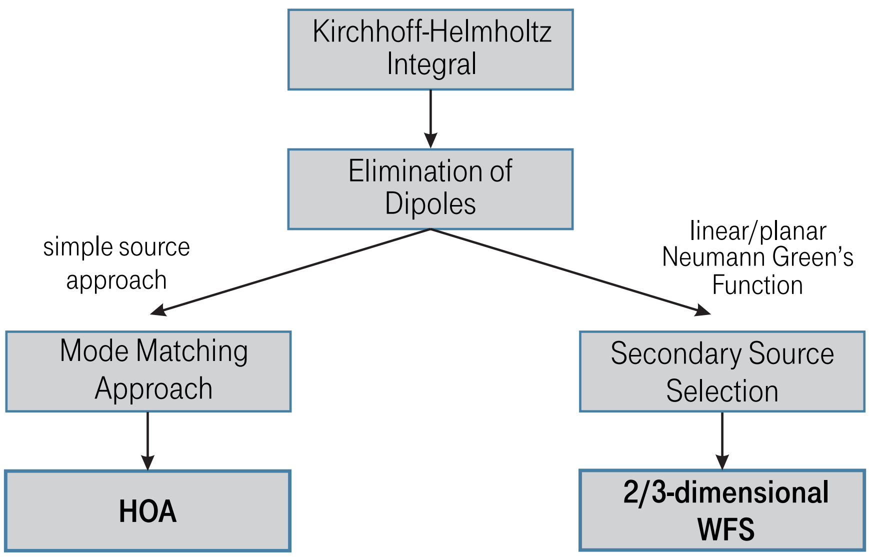
Application of the simple source approach for monopole only reproduction

$$P(\mathbf{x},\omega) = \oint_{\partial V} D_{\text{HOA}}(\mathbf{x}_0,\omega)\ G_0(\mathbf{x}|\mathbf{x}_0,\omega)\ dS_0$$

Fundamentals of HOA

- explicit solution of reproduction equation by method of moments
- choice of basis functions depends on underlying geometry
- solution is known to be not unique [Copley, 1967],
  no control over wave field for all frequencies (forbidden frequencies)
- exact reproduction in theory possible

Overview – Physical Foundations of HOA and WFS

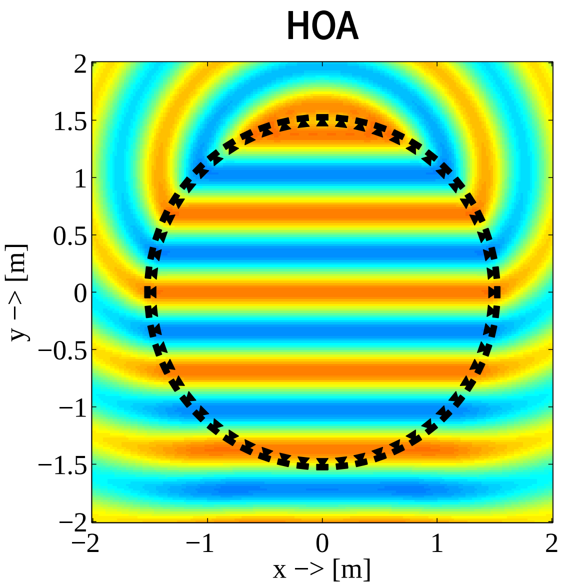


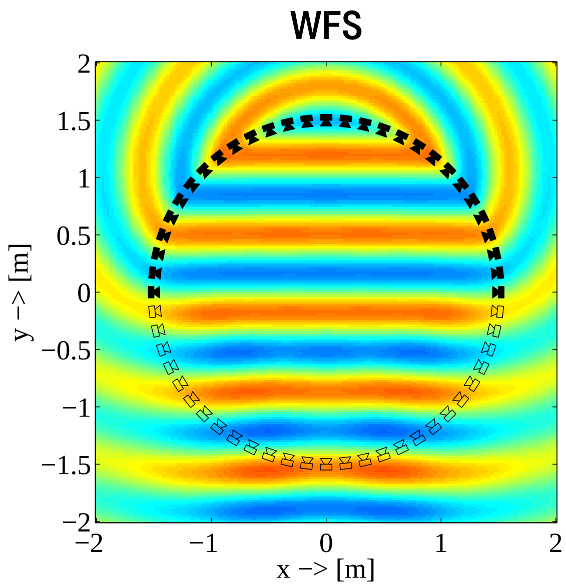
- analytic driving functions only for regular geometries
- exact reproduction within entire listening area possible

- analytic driving functions for arbitrary geometries
- exact reproduction only for linear/planar systems

Example – Reproduced Wave Field WFS/HOA

Reproduction of monochromatic plane wave with two-dimensional WFS/HOA

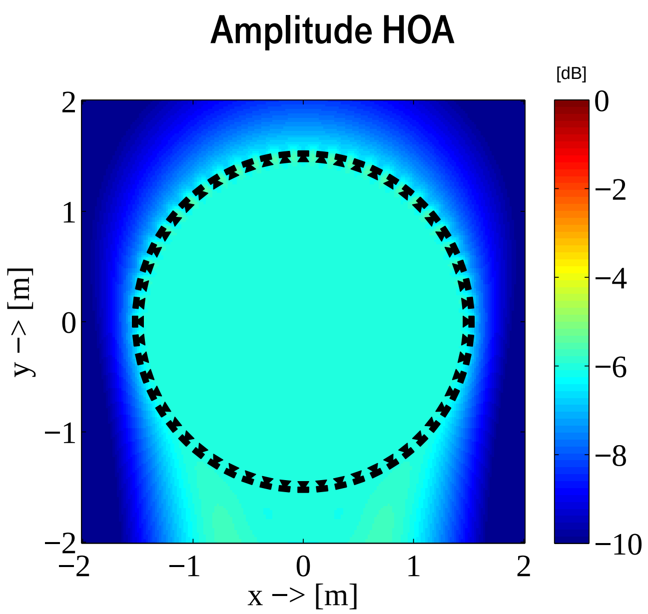


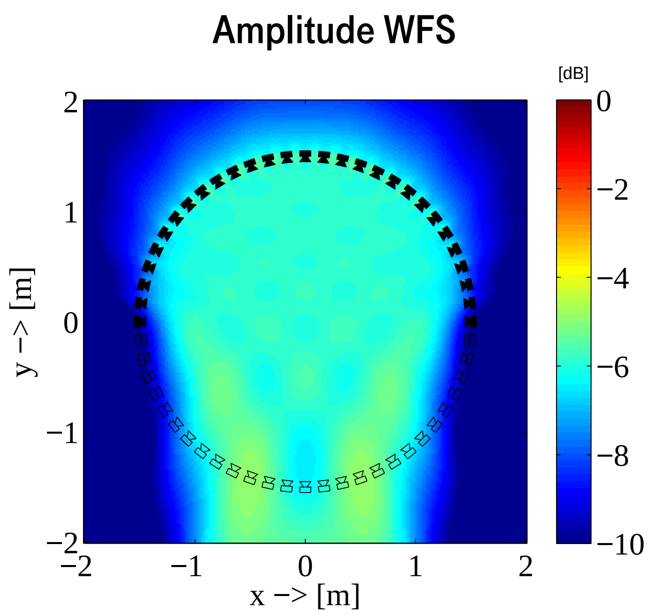


$[R = 1.50 \text{ m}, N = 56, f_{pw} = 500 \text{ Hz}, \alpha_{pw} = 270^{o}]$

Example – Reproduced Wave Field WFS/HOA

Reproduction of monochromatic plane wave with two-dimensional WFS/HOA





$[R = 1.50 \text{ m}, N = 56, f_{pw} = 500 \text{ Hz}, \alpha_{pw} = 270^{o}]$

Fourier Series Representation of Reproduced Wave Field

Unified description for a circular distribution of secondary line sources

$$P(\mathbf{x},\omega) = \int_0^{2\pi} D(\alpha_0, R, \omega)\, G_{0,2D}(\mathbf{x}-\mathbf{x}_0,\omega)\, d\alpha_0$$

Fourier series with respect to α

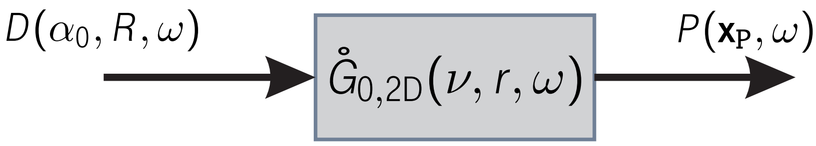
$$P(\mathbf{x},\omega) = \sum_\nu \mathring{P}(\nu, r, \omega)\, e^{j\nu\alpha}$$

$$\mathring{P}(\nu, r, \omega) = \mathring{D}(\nu, R, \omega) \cdot \mathring{G}_{0,2D}(\nu, r, \omega)$$

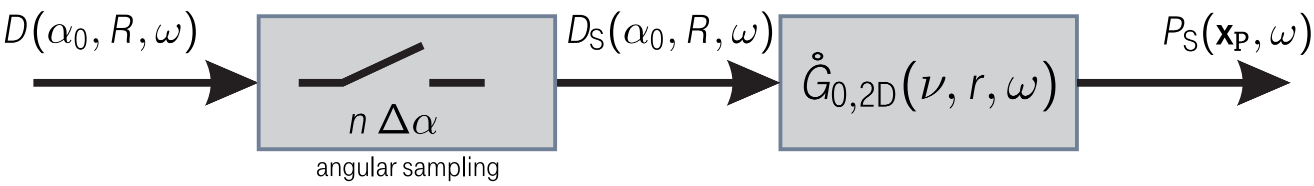
- Fourier series representation of (periodic) angular coordinate
- reproduced wave field is given by scalar multiplication in angular frequency domain

Spatial Sampling

Continuous distribution of secondary sources



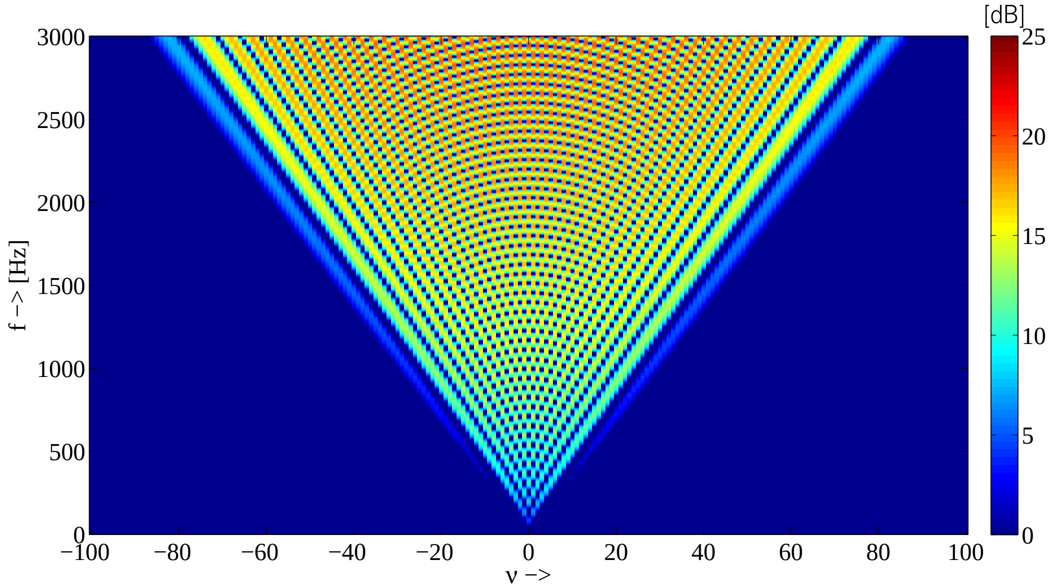
Spatially discrete distribution of secondary sources



- sampling of driving function models secondary source sampling
- repetition of angular spectrum of driving function due to angular sampling
- similar to sampling and interpolation process

Example – Characteristics of WFS Driving Function

Angular spectrum of a WFS driving function for a plane wave



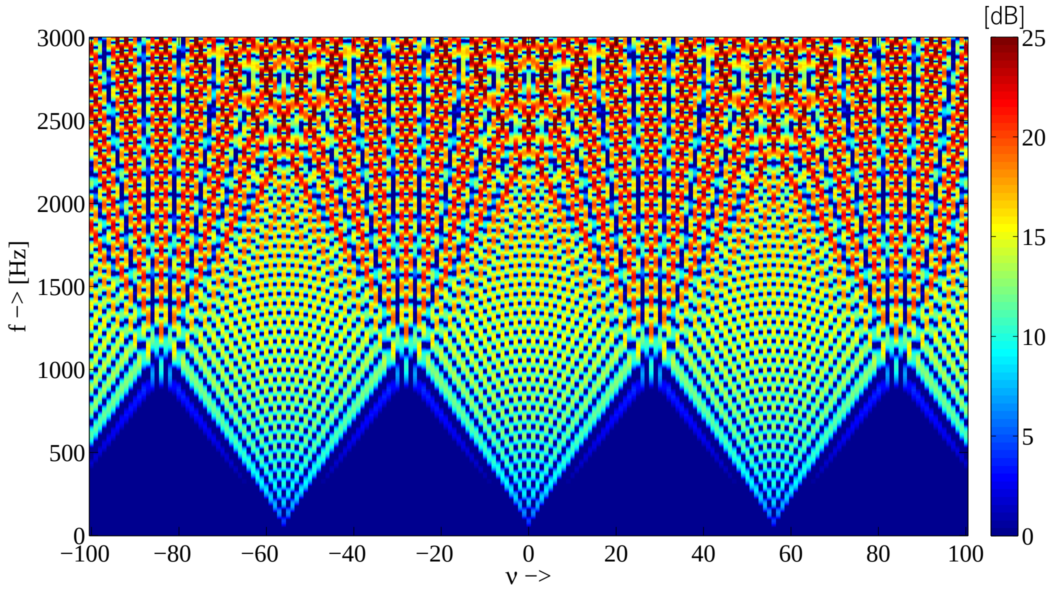
$[R = 1.50 \text{ m}, N = 56, \alpha_{\text{pw}} = 90^{\circ}]$

- driving function is not band-limited in the angular frequency domain

Introduction Physical Foundations Spatial Sampling Illustrative Example Conclusions

Example – Characteristics of WFS Driving Function

Angular spectrum of an **angular sampled** WFS driving function for a plane wave

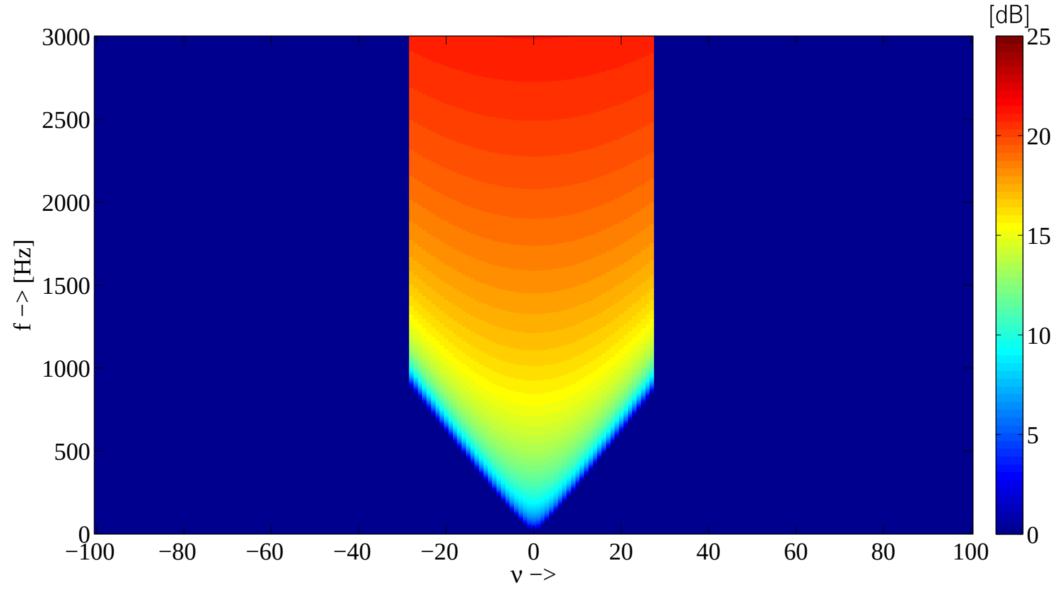


$[R = 1.50 \text{ m}, N = 56, \alpha_{\text{pw}} = 90^{\circ}]$

- angular sampling leads to repetition of angular spectrum and overlaps

Example – Characteristics of HOA Driving Function

Angular spectrum of a HOA driving function for a plane wave



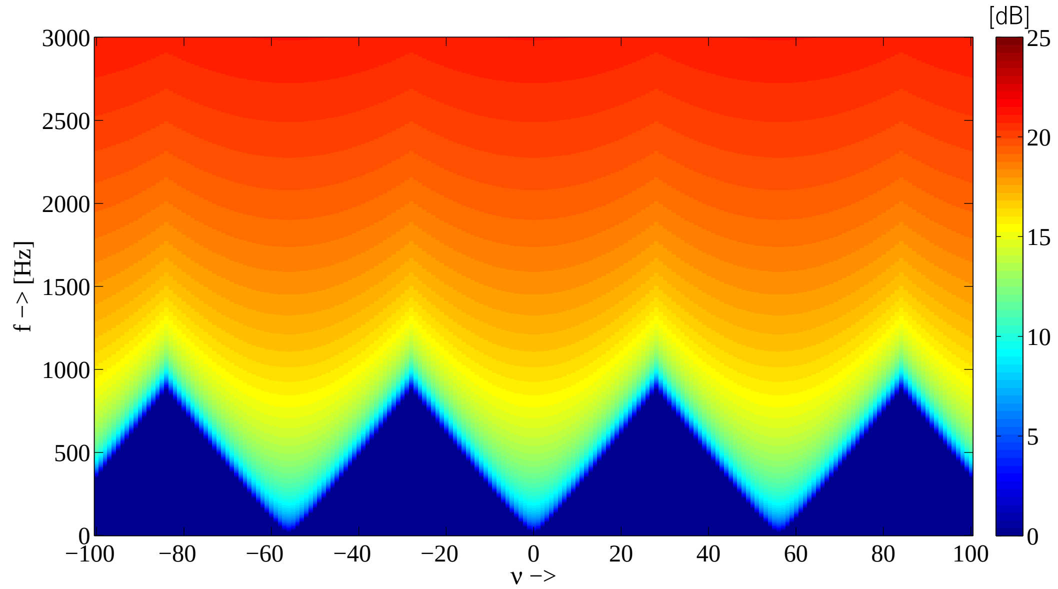
$[R = 1.50\text{ m}, N = 56, \alpha_{\text{pw}} = 90^{\circ}]$

- driving function is band-limited in the angular frequency domain

Introduction Physical Foundations Spatial Sampling Illustrative Example Conclusions

Example – Characteristics of HOA Driving Function

Angular spectrum of an **angular sampled** HOA driving function for a plane wave

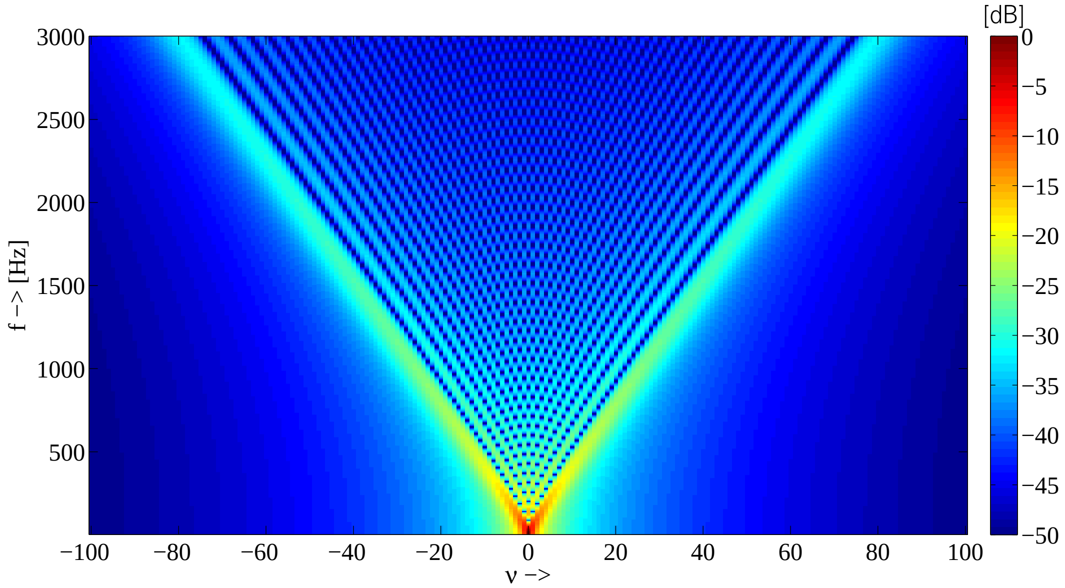


$[R = 1.50\text{ m}, N = 56, \alpha_{\text{pw}} = 90^{\circ}]$

- angular sampling leads to repetition of angular spectrum

Example – Secondary Source Spectrum

Angular spectrum of secondary line sources for circular system

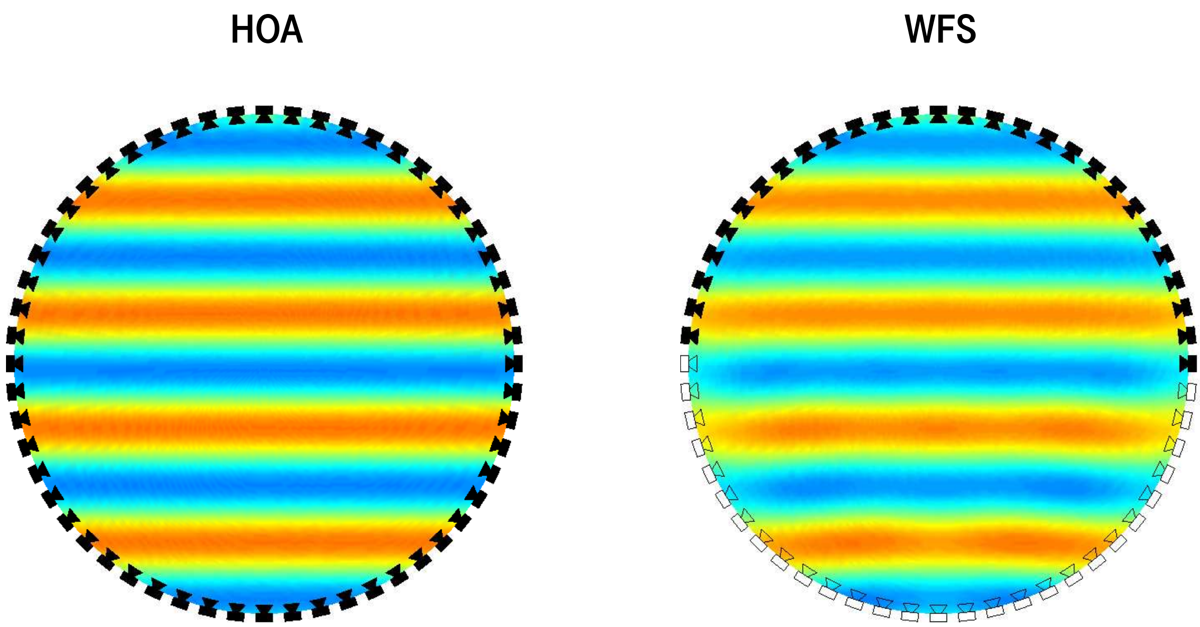


$[R = 1.50 \text{ m}, r = 1.50 \text{ m}, N = 56, \alpha_{\text{pw}} = 90^\circ]$

- spectrum is not band-limited in the angular frequency domain

Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{\text{pw}} = 500$ Hz)

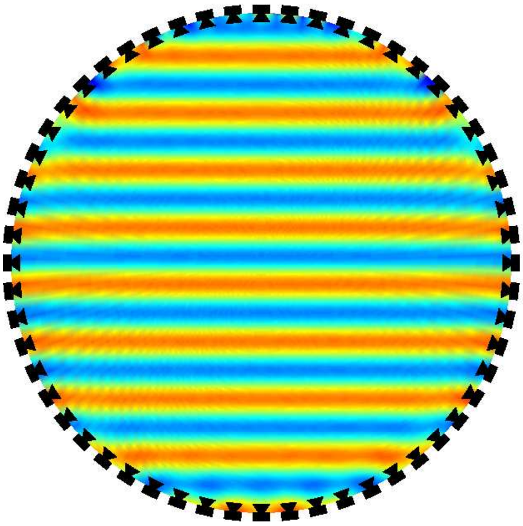


$[R = 1.50 \text{ m}, N = 56, \alpha_{\text{pw}} = 270^\circ]$

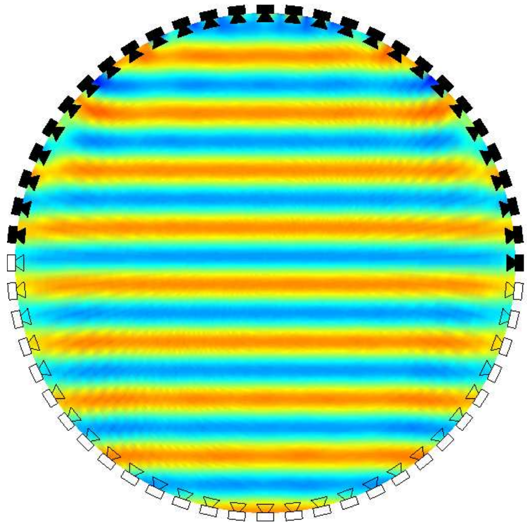
Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{pw} = 1000$ Hz)

HOA



WFS

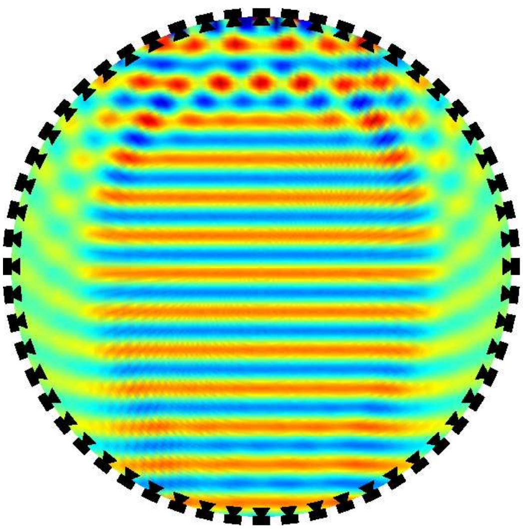


$[R = 1.50 \text{ m}, N = 56, \alpha_{pw} = 270^o]$

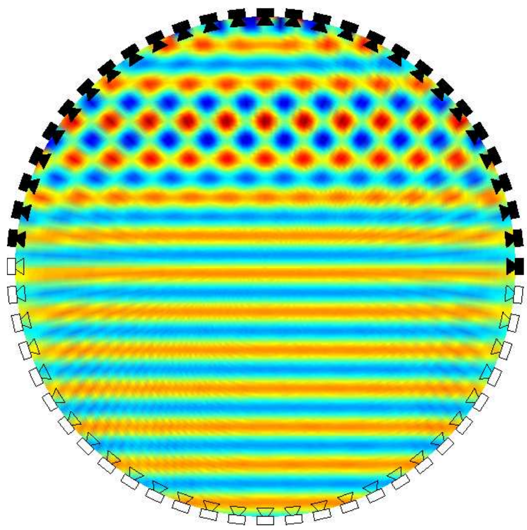
Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{pw} = 1500$ Hz)

HOA



WFS

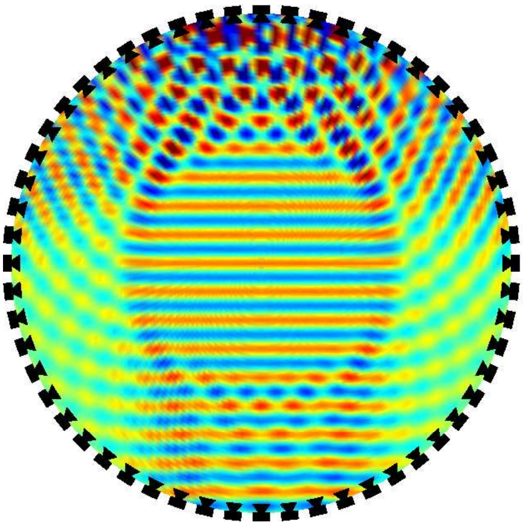


$[R = 1.50 \text{ m}, N = 56, \alpha_{pw} = 270^o]$

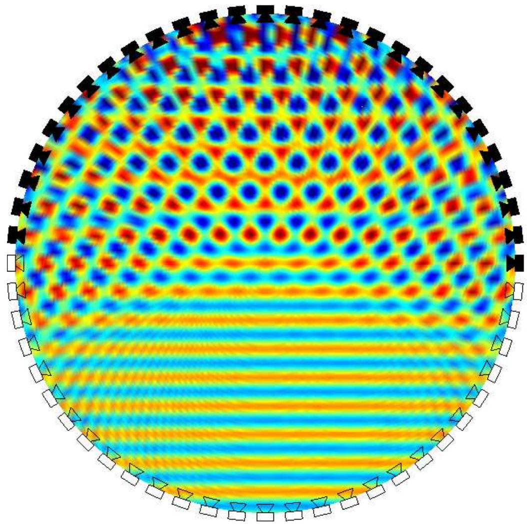
Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{\mathrm{pw}} = 2000$ Hz)

HOA



WFS

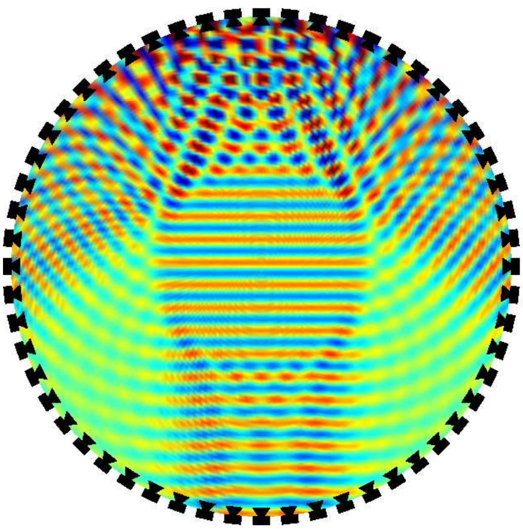


$[R = 1.50 \text{ m}, N = 56, \alpha_{\mathrm{pw}} = 270^{\circ}]$

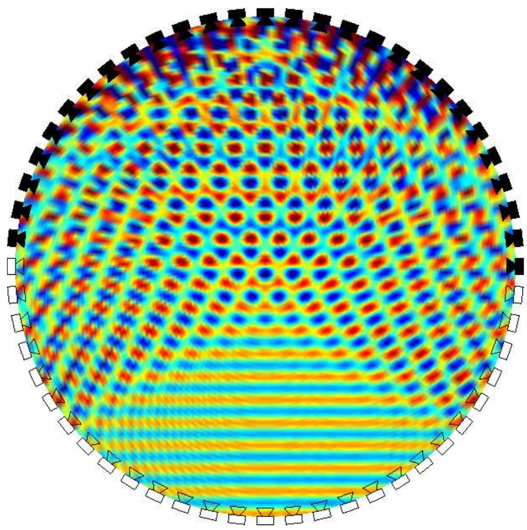
Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{\mathrm{pw}} = 2500$ Hz)

HOA



WFS

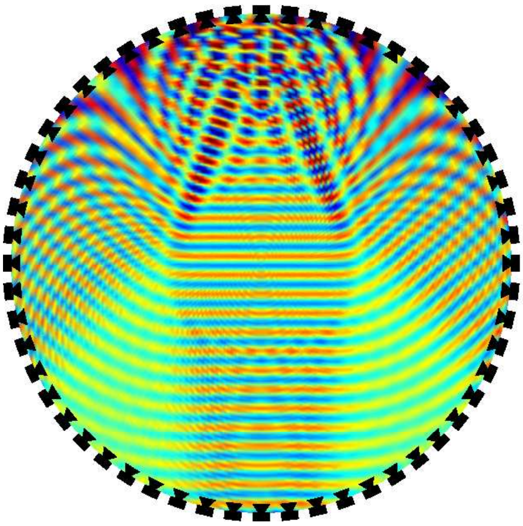


$[R = 1.50 \text{ m}, N = 56, \alpha_{\mathrm{pw}} = 270^{\circ}]$

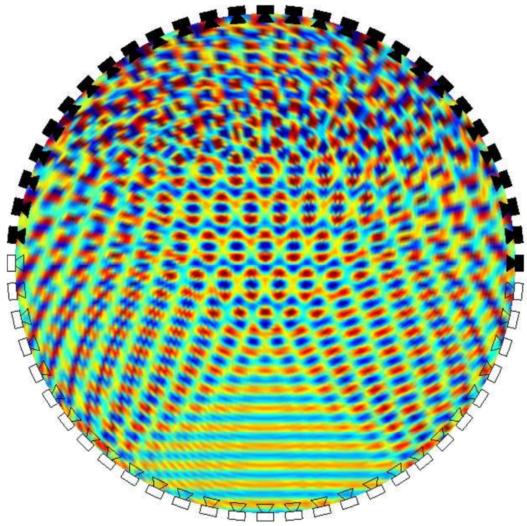
Example – Reproduced Wave Field HOA/WFS

Reproduction of monochromatic plane wave ($f_{pw} = 3000$ Hz)

HOA



WFS

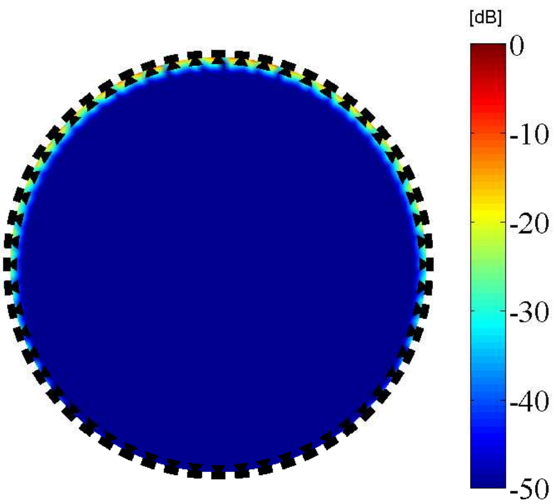


[$R = 1.50$ m, $N = 56$, $\alpha_{pw} = 270^o$]

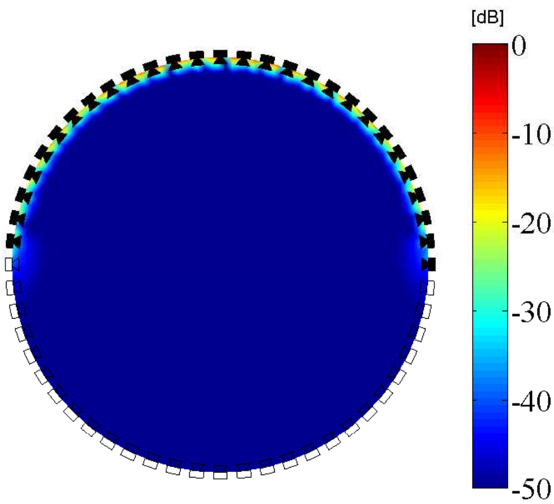
Example – Reproduced Aliasing to Signal Ratio

RASR for reproduction of a bandlimited plane wave ($b_{pw} = 500$ Hz)

HOA



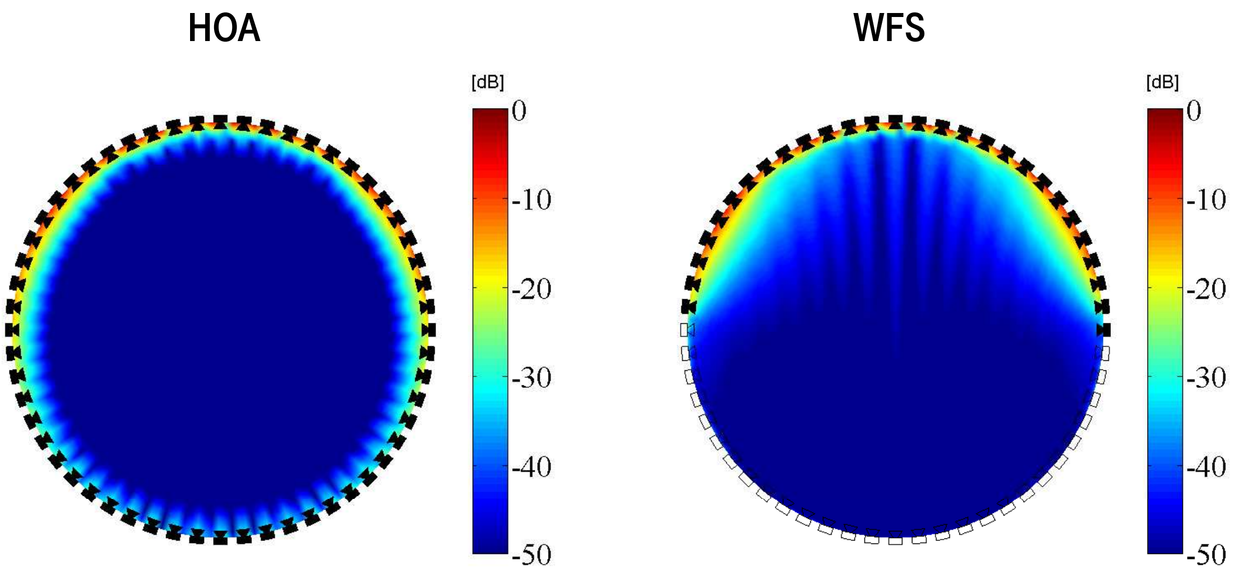
WFS



[$R = 1.50$ m, $N = 56$, $\alpha_{pw} = 270^o$]

Example – Reproduced Aliasing to Signal Ratio

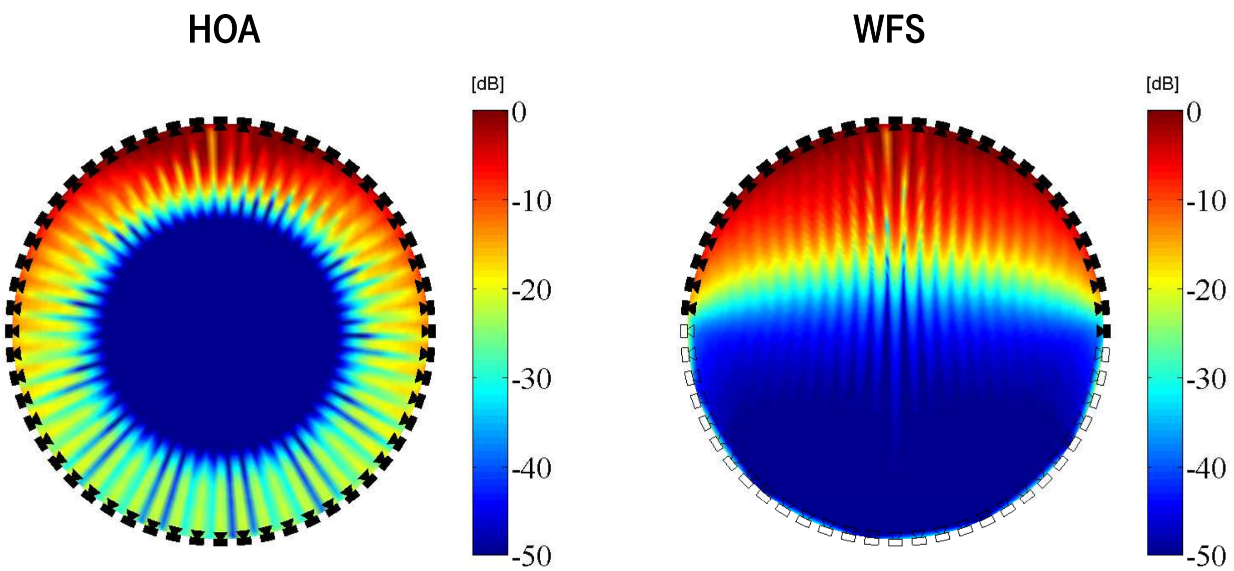
RASR for reproduction of a bandlimited plane wave ($b_{\mathrm{pw}} = 1000$ Hz)



$[R = 1.50\ \mathrm{m},\ N = 56,\ \alpha_{\mathrm{pw}} = 270^{\circ}]$

Example – Reproduced Aliasing to Signal Ratio

RASR for reproduction of a bandlimited plane wave ($b_{\mathrm{pw}} = 1500$ Hz)

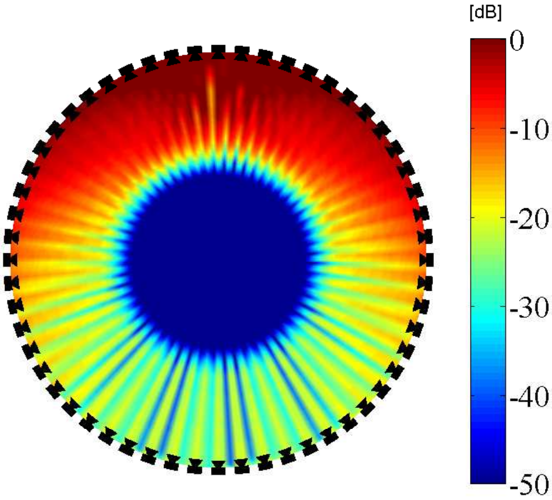


$[R = 1.50\ \mathrm{m},\ N = 56,\ \alpha_{\mathrm{pw}} = 270^{\circ}]$

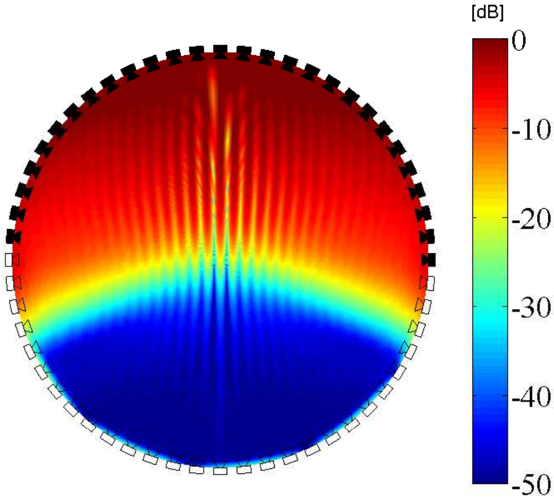
Example – Reproduced Aliasing to Signal Ratio

RASR for reproduction of a bandlimited plane wave ($b_{pw} = 2000$ Hz)

HOA



WFS

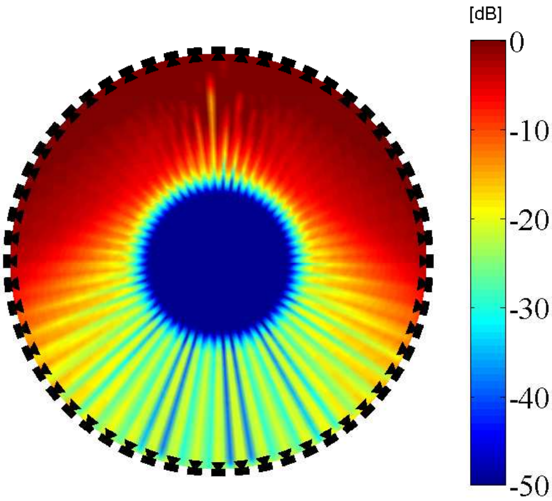


$[R = 1.50 \text{ m}, N = 56, \alpha_{pw} = 270^\circ]$

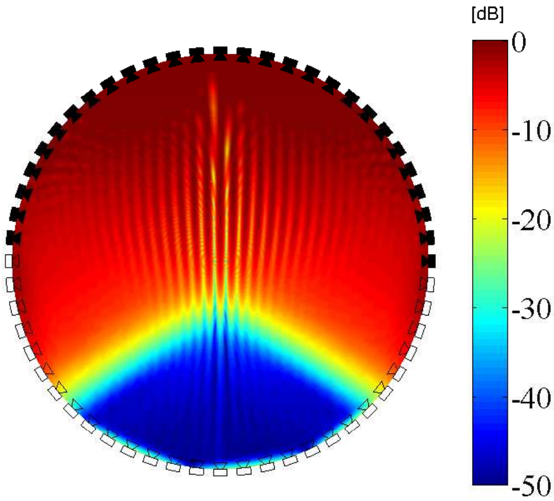
Example – Reproduced Aliasing to Signal Ratio

RASR for reproduction of a bandlimited plane wave ($b_{pw} = 2500$ Hz)

HOA



WFS

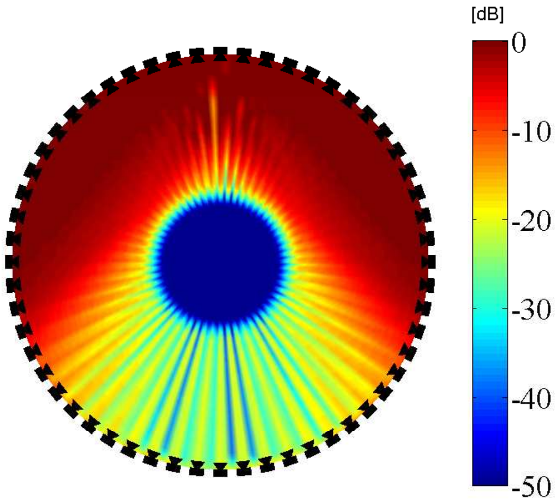


$[R = 1.50 \text{ m}, N = 56, \alpha_{pw} = 270^\circ]$

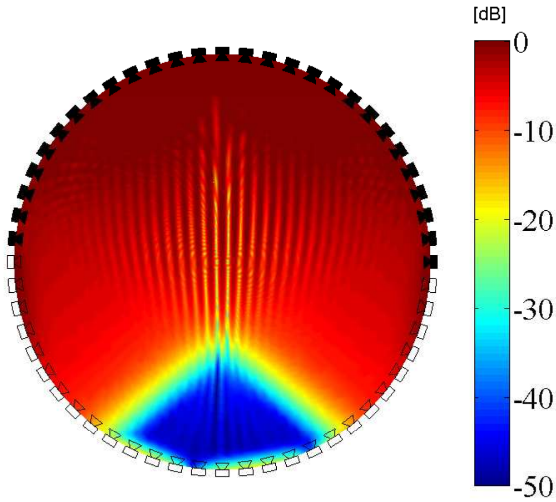
Example – Reproduced Aliasing to Signal Ratio

RASR for reproduction of a bandlimited plane wave ($b_{\mathrm{pw}} = 3000$ Hz)

HOA



WFS



[$R = 1.50$ m, $N = 56$, $\alpha_{\mathrm{pw}} = 270^{\circ}$]

Summary and Conclusions

This paper presents a comparison of the physical foundations of WFS and HOA, and a detailed investigation of the spatial sampling process and its artifacts.

Main findings for WFS

- analytic driving functions for arbitrary geometries
- no exact reproduction for non-linear/planar geometries
- almost alias-free area is not at a constant position
- sampling artifacts exhibit irregular structures

Main findings for HOA

- analytic driving functions only for simple geometries available
- exact reproduction within the listening area in theory possible
- suffers from non-uniqueness (forbidden frequencies)
- sampling artifacts exhibit quite regular structures