Panpot Laws for Multispeaker Stereo

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PANPOT LAWS FOR MULTISPEAKER STEREO

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Abstract

Unlike the well-accepted constant-power sine/cosine law for 2-speaker stereo, there is no well accepted panpot law for frontal-stage stereo systems using three or four loudspeakers. Various proposed laws, including that of Bell Telephone Laboratories and one based on perfect reproduction of velocity are considered, and it is shown that optimal stereo imaging is given by a compromise law that is almost independent of the angular width of the speaker layout.

1. INTRODUCTION

While multispeaker frontal-stage stereo reproduction has been applied to reproduction in auditoria since 1933 [1], especially to large-screen film applications using between 3 and 5 speakers [2], it is only recently that domestic applications have come to the fore. With High Definition Television (HDTV), most workers agree that either three or four independent loudspeaker signals are needed to give an adequately stable front-stage stereo image to match that on the screen across a listening/viewing area [3-7].

However, relatively little attention has been paid to the design of panning methods that optimise stereo localisation quality via such domestic scale systems. It is widely recognised that the smaller sound-travel delays in domestic reproduction as compared to auditorium reproduction somewhat change any system optimisation for stereo, with the domestic situation being capable of giving more critical quality results. The problem of finding panpot laws had been tackled as early as 1933 [1] for auditorium reproduction, and by the 1950's, the use of both amplitude and time-delay panning for film use was well established [2]. However, anomalies due to large interspeaker time delays that are largely masked in the auditorium environment become audible in smaller-scale domestic environments, whereas the potential sharpness of phantom images is correspondingly improved.

The aim of the present paper is to find and analyse optimal panpot laws based on amplitude panning techniques for domestic use, although the laws discussed are also expected to be useful in an auditorium environment. The optimisation of an amplitude panning law generalises the discovery by Blumlein in 1931 [8] of the sine/cosine panning law for 2-speaker stereo, and is based on a generalisation of his theoretical psychoacoustic methods.

The aim of a good panpot law is to take monophonic sounds, and to give each one amplitude gains, one for each loudspeaker, dependent on the intended illusory directional localisation of that sound, such that the resulting reproduced sound provides a convincing and sharp phantom illusory image. Such a good panpot law should provide a smoothly continuous range of image directions for any direction between those of the two outermost loudspeakers, with no "bunching" of images close to any one direction or "holes" in which the illusory imaging is very poor.

Moreover, a good panpot law should give sound images that satisfy as many as possible different auditory localisation mechanisms as possible for a listener at an ideal stereo seat, so that the localisation is reliable, natural, and gives low listening fatigue. In addition, the localisation should remain as stable as possible (relative to the angular directions of the loudspeakers) with movement of a listener across a wide listening area.

There is, of course, no problem in creating an "illusion" of sounds from the actual directions of the loudspeakers - one simply feeds a sound only to that loudspeaker! However, the requirement of a smoothly continuous panpot law poses considerable difficulties, since one needs to be able also to create the illusion of phantom images of good localisation quality in directions close to but not at the speaker directions. If one uses a "discrete" panpot law in which a sound intended for a speaker direction is fed only to that loudspeaker,then it can be shown that near-speaker directions are pulled into that speaker, creating what has been termed a "detent" effect [7,9], which means that the subjective effect of the law is not smoothly continuous.

In order to optimise panpot laws, one needs a good theoretical model for the psyschoacoustics of directional localisation. The model used in this paper is based on objective physical quantities describing the reproduced sound field at the listener, i.e. pressure, acoustic velocity, energy and the sound-intensity vector. The theory used has previously been published in a number of places [10-13], and is found to provide a good basis for optimising the directional psychoacoustics of domestic sound reproduction systems.

Because the theoretical psychoacoustic models for the phantom image localisation quality of images are central to the understanding of the design of panpots, we feel that it is necessary to repeat the relevant parts of the theory and interpretation in this paper, despite adequate previous accounts in [10-13].

It is necessary to have a clear idea of the design aims in designing "optimal" panpot laws. A vague statement that "localisation quality should be good" is not adequate, since design involves a number of tradeoffs between possibly conflicting factors, and, as we shall see, a design optimised for only one of the relevant factors may be very badly behaved as regards other important factors. We can list here some of the relevant factors: -

(i) For listeners at an ideal stereo seat, the sound localisation direction according to low-frequency interaural phase and mid/high-frequency interaural amplitude auditory localisation mechanisms should be substantially identical, so as to avoid image blurring. This localisation identity should ideally be achieved by a frequency-independent panning law so that both classes of auditory localisation mechanisms agree across a common overlapping frequency range.

(ii) The apparent image localisation should be reasonably stable with listener movement across the listening area. More precisely, the degree of apparent image movement relative to the directions of the loudspeakers, should be minimised. Note that this requirement is not the same as requiring that the absolute sound directions should be stable - i.e. one requires that the virtual sound source distance under listener movement be that of the speakers rather than being infinite.

(iii) The apparent image localisation should be as stable as possible under rotation of the orientation of the listener.

(iv) As the listener moves across the listening area, any image movement should be such as to minimise geometric distortion of the stereo stage, i.e. as far as possible, the degree and direction of movement of different parts of the image should be similar so as to cause primarily only an overall displacement of the total sound stage.

(v) The panpot law should ideally be frequency-independent such that reduction of the sound to mono or to stereo using fewer loudspeakers does not suffer from unpleasant frequency- and position-dependent colouration artifacts. This is particularly important for compatible use with hierarchies of encoding and decoding systems for transmission systems, see [14].

(vi) The panpot law should avoid large changes of image localisation quality with small changes in apparent image position, as otherwise the ears will not be able to adapt to any localisation imperfections to make maximum sense of the available cues.

(vii) the actual panning control should give smooth and uniform movement of images as the control is moved.

(viii) Finally, the panpot law should approximate a constant power behaviour, with equal energy gain fed into the listening room from all panpot positions.

In this paper, we shall show that it is possible to devise panpot laws for 3- and 4-speaker stereo that meet these requirements to a high degree, but that "discrete" and "pairwise" panpot laws of the kind often used fail to meet requirements (i), (iv), (vi) and (vii) above.

While the "optimum" panpot law meeting requirement (i)-(viii) above varies with the angular width of the stereophonic speaker layout, this variation is in practice sufficiently small that a single compromise panpot law may be used for all reasonable angular widths.

The study of panpot laws is particular important in studying the compatibility of different transmission and reception modes for hierarchical systems of multispeaker stereo (see [14]). While in ref. [14], we showed that the upward compatibility (i.e. the reception via more loudspeakers than used for the transmission) of such systems could be ensured without any detailed assumptions about the method of panning used, the same is not true of downward compatibility (i.e. the recption and reproduction via fewer speakers than used for transmission). The reason for this is that no actual information is lost when more loudspeakers are used, whereas the use of fewer loudspeakers means that some signal components are nulled out; the magnitude and importance of such cancelled signal components varies greatly according to the panpot law used, and this has a great influence on downward compatibility. Clearly, the downward compatibility performance of any hierarchical system of transmitting multispeaker stereo should be optimised not for an arbitrary ad-hoc method of panning such as pairwise panning between adjacent speaker pairs, but for an optimised panpot law satisfying the requirements (i) to (viii) above.

Thus we see that a study of optimal panpot laws is relevant not only for the design of multispeaker mixing desks, but also for the design and optimisation of hierarchical systems of handling stereo intended for different numbers of loudspeakers in a mutually compatible fashion.

In order to illustrate the advantages of optimised panpot laws, this paper also analyses the behaviour of a variety of plausible or proposed suboptimal laws, such as the early Bell law [1] and a law [7] designed for optimum interaural phase localisation at low frequencies.

It is likely that many balance and recording engineers may find it implausible that an optimum panpot law is <u>not</u> "discrete", since it is evident that the best illusion of localisation from the direction of a loudspeaker is obtained if only that loudspeaker is fed with sound. It is only when one considers the relationship of that sound position to all the other "phantom" sound image directions, as in requirements (i), (iv), (vi) and (vii) above, that a problem becomes apparent. As noted in [7], good stereo with high intelligibility and low listener fatigue preferably has different sounds panned to at least slightly different stereo positions, and in such recordings or mixes, it is not acceptable to optimise isolated image directions at the expense of grossly degrading all nearby image positions.

2. PSYCHOACOUSTIC THEORY

The quality of localisation of phantom sound images can conveniently be described in terms of objective physical quantities at the ideal listening position. The details of this theory have previously been published in refs. [10-13], and we simply summarise the aspects we require here, without the detailed justification given in those references.

Consider, as shown in figure 1, n loudspeakers placed, in different angular directions, at equal distances from an ideal listening position. Rectangular axes are used with the x-axis pointing forwards and the y-axis leftwards, and angular directions are measured as angles θ measured anticlockwise (i.e. towards the y-axis) from due front (i.e. the x-axis). The i'th loudspeaker is assumed to be fed a panned monophonic signal with a gain G_i as shown, and to lie at an angular direction $\hat{\theta}_i$ as shown.

The localisation theories are all based on the following idea (see figure 2). Suppose that each speaker emits a sound "magnitude" g_i, whose physical nature we do not specify precisely for the moment. Then the total sound "magnitude" for a central listener is simply the sum

$$\sum_{i=1}^{n} g_i \tag{1}$$

of the magnitudes emitted by the individual loudspeakers. The vector direction associated with these emitted magnitudes can be determined by drawing a vector of length g_i pointing at each loudspeaker, as shown in figure 2, and taking their vector sum, which has respective x- and y- axis components

$$\sum_{i=1}^{n} g_i \cos\hat{\theta}_i \tag{2}$$

and

$$\sum_{i=1}^{n} g_i \sin\hat{\theta}_i \quad . \tag{3}$$

This summed vector is then "normalised" in length by dividing by the total sum of magnitudes (1). This normalised vector has respective x- and y-axis components of the form

$$r\cos\theta \;=\; \left(\sum_{i=1}^{n} g_i \cos\hat{\theta}_i\right) \Big/ \left(\sum_{i=1}^{n} g_i\right) \tag{4}$$

and

$$r\sin\theta \;=\; \left(\sum_{i=1}^{n} g_i \sin\hat{\theta}_i\right) \Big/ \left(\sum_{i=1}^{n} g_i\right) , \tag{5}$$

where $r \geqslant 0$ is the vector length of the normalised vector and θ is the angular direction in which the normalised vector points. For single sound sources, θ is the direction of the sound source and $r = 1$. Depending on the choice of the "magnitude" g_i, in the general case θ is a measure of the perceived illusory sound direction when the listener faces the apparent sound source, and the deviation of r from the ideal value 1 determines the degree of instability of sound localisation with listener rotation or movement.

If the choice of "magnitude" g_i is the actual gain G_i with which a sound is fed to a speaker, then the sum (1) is the total pressure gain at the listener and the vector components (2) and (3) are components of the vector velocity gain at the listener. In this case, the vector length

$$r = r_V \tag{6}$$

and the vector direction

$$\theta = \theta_V \tag{7}$$

are respectively termed the <u>velocity magnitude</u> and the <u>velocity direction</u> or <u>Makita direction</u> (after the work of Makita [15]). The velocity or Makita direction is the apparent sound localisation direction according to low-frequency interaural phase localisation theories (particularly apt below around 700 Hz) when the listener faces the apparent sound source, and the velocity magnittude r_V describes the degree of phantom image movement according to interaural phase localisation theories as the listener's head is rotated; if $r_V < 1$, the apparent image rotates in the same direction as the head, whereas if $r_V > 1$, the apparent image rotates in the opposite direction.

If, instead we choose the "magnitudes" g_i to equal the squares of the gains G_i, i.e. put

$$g_i = G_i^2 , \tag{8}$$

then the normalised vector length

$$r = r_E \tag{9}$$

and the vector direction

$$\theta = \theta_E \tag{10}$$

are respectively termed the <u>energy vector magnitude</u> and the <u>energy vector direction</u>. In this choice (8), the sum (1) is the total energy gain into the room and the vector with components (2) and (3) is the vector sound-intensity gain. The direction θ_E is used to determine apparent sound direction for listeners facing the apparent sound source for frequencies between around 700 Hz and 3.5 kHz, although it will be realised that these frequencies are "fuzzy" and that there is in practice overlap in the frequency ranges at which θ_V and θ_E are used to determine localisation. Also, it can be shown that if the signals from the different loudspeakers are phase incoherent (e.g. due to different speaker distances), then low-freuency interaural phase theories then predict that the vector magnitude and velocity directions are equal to r_E and θ_E respectively, so that the energy localisation parameters r_E and θ_E also describe interaural phase localisation in the phase-incoherent case such as non-central listening.

In the case (8) where localisation is determined by energy gains, which we term the energy vector theory, r_E can never exceed one, and only equals one when a sound is emitted by a single loudspeaker. The value of r_E determined for a central listener turns out to provide a good predictor of the degree of image movement as listeners move away from a central listening position. It is useful both above 700 Hz and also at much lower frequencies since very noncentral listeners are in the phase-incoherent region. The following rule of thumb holds quite well: The degree of angular movement of phantom images relative to the apparent speaker directions caused by any given degree of listener movement is proportional to $1-r_E$. Thus $r_E = 0.95$ causes about one third of the degree of image movement as does $r_E = 0.85$.

This rule of thumb works quite well both for localisation of continuous sounds and for transient sounds, although it is found that the actual absolute amount of image movement for transient sounds is generally larger than for continuous sounds. This rule of thumb seems to predict rather well the change in the size of listening are using different numbers of loudspeakers for HDTV reported by Theile and others [3,4]. The $1-r_E$ proportionality rule for image movement predicts that, for a given acceptable amount of image movement, and for speaker layouts covering the same total sector of directions, 3-speaker stereo will have a listening area around 4 times as wide as 2-speaker stereo, 4-speaker stereo will have a listening area 9 times as wide and 5-speaker stereo will have a listening area 16 times as wide, approximately.

The two velocity theory localisation parameters r_V and θ_V, and the two energy theory localisation parameters r_E and θ_E are termed the localisation parameters for the gains G_i fed to a speaker layout, and between them provide good predictors for localisation quality and stabilty of images under listener movement.

A particular requirement for high-quality sound images for listeners in the ideal central stereo seat is that the two theories should agree as to the basic sound localisation direction, i.e. it is desirable that

$$\theta_V = \theta_E \tag{11}$$

especially in that frequency range in which both theories have applicability. The requirement (11) is met by conventional 2-speaker stereo only for three sound positions, i.e. left, centre and right, and fails at other positions such as half-left or half-right. This causes a long-known discrepancy of localisation cues which causes unsatisfactory imaging and listener fatigue.

One of the primary aims in designing panpot laws for 3- or more loudspeakers, as it was for Ambisonics [11,12], is to ensure that equation (11) holds, at least to a good approximation. Within this constraint, we would like to get r_E as large as possible, in order to maximise image stability, and also try to get r_V reasonably close to one - although we shall see that (unlike the Ambisonic surround-sound case), it is unwise to place excessive weight on the last criterion.

3. OPTIMUM 3-SPEAKER PANPOT LAW

In figure 3, we show a 3-speaker stereo layout, with respective left, centre and right speakers denoted L_3, C_3 and R_3 placed at equal distances from a central listener at respective azimuth angle directions θ_3, 0 and $-\theta_3$. We seek to determine gains with which a sound can be fed to the three loudspeakers such that equation (11) holds. We shall derive an exact analytic solution, which depends on the angular width $2\theta_3$ of the 3-speaker layout.

Denote the three gains with which a mono sound is panned to the three loudspeakers by the same symbols, L_3, C_3 and R_3 as used for the loudspeakers themselves. Then, by equations (4) and (5), the velocity vector and energy vector localisation azimuths θ_V and θ_E respectively satisfy the equations:

$$\tan\theta_V = \frac{(L_3 - R_3)\sin\theta_3}{C_3 + (L_3 + R_3)\cos\theta_3} \tag{12}$$

and

$$\tan\theta_E = \frac{(L_3^2 - R_3^2)\sin\theta_3}{C_3^2 + (L_3^2+R_3^2)\cos\theta_3} \quad . \tag{13}$$

If equation (11) is satisfied, i.e. $\theta_V = \theta_E$, then the right hand sides of equs. (12) and (13) are equal, and this implies either that

$$L_3 = R_3 \tag{14}$$

which gives a central sound direction, due to the factor $L_3 - R_3$ common to equs. (12) and (13), or else that

$$(L_3 + R_3)(C_3 + (L_3 + R_3)\cos\theta_3) = C_3^2 + (L_3^2 + R_3^2)\cos\theta_3 \tag{15}$$

by removing the common factor $(L_3 - R_3)$ and multiplying by the denominators in (12) and (13). Equation (15) must be satisfied by a panpot covering non-central positions for which equ. (11) is satisfied, and after further algebraic simplification gives

$$C_3(C_3 - L_3 - R_3) = (2\cos\theta_3)L_3R_3 \; . \tag{16}$$

From equation (12), we also have that

$$C_3 = -(L_3 + R_3)\cos\theta_3 + (L_3 - R_3)\frac{\sin\theta_3}{\tan\theta_V} \quad . \tag{17}$$

In order to derive a panpot law, we must in some way normalise the overall gain of the signals, since an overall change of all gains does not affect the stereo localisation parameters. Initially, we choose an ad-hoc normalisation that maximises analytic simplicity, and only at the end of the analysis will we introduce our desired normalisation, which is that which gives constant power gain.

Thus we shall put

$$L_3 = 1 + \varepsilon \quad \text{and} \quad R_3 = 1 - \varepsilon \; , \tag{18}$$

so that equ. (15) becomes:

$$C_3^2 = 2C_3 + (2\cos\theta_3)(1 - \varepsilon^2) \tag{19}$$

and equation (17) becomes

$$C_3 = -2\cos\theta_3 + 2\varepsilon\frac{\sin\theta_3}{\tan\theta_V} \quad . \tag{20}$$

Substituting equ. (20) into equ. (19) yield the following quadratic equation in ε :-

$$\left[2\frac{\sin^2\theta_3}{\tan^2\theta_V} + \cos\theta_3\right]\varepsilon^2 - 2(1+2\cos\theta_3)\frac{\sin\theta_3}{\tan\theta_V}\varepsilon + (1+2\cos\theta_3)\cos\theta_3 = 0 \tag{21}$$

whose solution is given by

$$\varepsilon = \frac{A(1+2\cos\theta_3) \pm \sqrt{[A^2 - \cos^2\theta_3](1+2\cos\theta_3)}}{2A^2 + \cos\theta_3} \; , \tag{22}$$

where

$$A = (\sin\Theta_3)/\tan\Theta_V \ . \tag{23}$$

Substituting the computed value of ε from equs. (22) and (23) into equs. (18) and (20) gives the required panpot law, except that we still need to provide the desired gain normalisation.

The gain normalisation for a total energy gain of unity is provided by diving by the square root of the overall energy gain, giving normalised panpot gains

$$\hat{L}_3 = L_3/(L_3{}^2+C_3{}^2+R_3{}^2)^{\frac{1}{2}} \tag{24a}$$

$$\hat{C}_3 = C_3/(L_3{}^2+C_3{}^2+R_3{}^2)^{\frac{1}{2}} \tag{24b}$$

$$\hat{R}_3 = R_3/(L_3{}^2+C_3{}^2+R_3{}^2)^{\frac{1}{2}} \ . \tag{24c}$$

There are two solutions to the 3-speaker panpot law satisfying equ. (11), namely those associated with the two choices of the $\pm$ sign in equ. (22). One chooses that sign for which r_E is largest in order to ensure the most stable sound localisation; for $\Theta_V = \Theta_E > 0$, this choice is + and for $\Theta_V = \Theta_E < 0$, this choice is - .

In order to assess the performance and behaviour of this "optimum" panpot law, it is also necessary to compute, using equs. (4) and (5), the values of r_V and r_E for each image localisation angle $\Theta = \Theta_V = \Theta_E$, in order to ensure that no poor image localisation behaviour that is unacceptable occurs.

The results of these calculations are shown in tables 1 and 2 for the two cases of a speaker layout with $\Theta_3 = 30^\circ$ and with $\Theta_3 = 45^\circ$, where we have computed the 3 channel gains for localisation angles Θ expressed as a fraction k of Θ_3, i.e.

$$\Theta = k\Theta_3 \ , \tag{25}$$

in order to provide comparisons of the panpot law for different values of Θ_3. Tables 1 and 2 only show the left half of the stereo stage since the right half is mirror-symmetric. We have given increased resolution near k = 1 in order to reveal the fine detail of the behaviour of the panpot law near the two edges of the stereo stage, since this behaviour departs, albeit not by a large amount, from the ideal of very smooth behaviour. Figure 4 shows the panpot law gains for L_3, C_3 and R_3 for different angular positions for $\Theta_3 = 30^\circ$; the graph for $\Theta_3 = 45^\circ$ is similar. Figures 5 and 6 show respectively the values of the 4 localisation parameters for different angular positions for $\Theta_3 = 30^\circ$ and $\Theta_3 = 45^\circ$. It will be seen that not only is the localisation the same according to both theories (which, of course, is what we designed the law for), but the value of $1 - r_E$ does not vary very much across the central 75% of the stereo stage. This is very desirable behaviour, since it means that any image movement with listener position will affect most sounds in the stereo stage to approximately the same degree, resulting in an overall sideways shift of the stereo stage without geometric distortion within the stage. The only exception are sounds near the left and right loudspeakers, for which $1 - r_E$ approaches zero, which remain "tied" to the two speaker locations as the listener position is changed.

The $\theta_3 = 30^\circ$ and $\theta_3 = 45^\circ$ optimal 3-speaker panpot laws given in tables 1 and 2 are not exactly identical, but they differ by an amount sufficiently small that in practice one of these laws will give good localisation quality via the other speaker layout. Thus, even in applications where different layout angles are used, say $\theta_3 = 30^\circ$ for HDTV use and say $\theta_3 = 45^\circ$ for audio-only use, a single panpot law, say that of table 1 and figure 4, can be used for both.

From fig. 4 and tables 1 and 2, it will be noted that the gains change very rapidly close to the extreme left and right extreme positions, especially beyond $k = 0.95$. In practice, a much smoother panpot law can be achieved if the travel of the control is restricted say to the range $-0.95 \leq k \leq 0.95$, without any noticeable loss of stereo width..

The price paid for the desirable equality of θ_V and θ_E at all positions is that central sounds cross-talk onto the two outer loudspeakers. Table 3 shows the degree of centre-to-edge crosstalk in dB of an optimal panpot at the centre panpot setting for various values of the layout half-angle θ_3, computed by putting $\varepsilon = 0$ in equ. (19).

Although this cross-talk reduces central image stability, the value of $1 - r_E$ for such central sounds is still smaller than for sounds at azimuth angle $\frac{1}{2}\theta_3$, which have $r_E = \cos\frac{1}{2}\theta_3$, and which are panned by equal feeds to the L_3 and C_3 speakers with no feed to the R_3 speaker. Since one cannot have lower r_E at azimuth $\frac{1}{2}\theta_3$ than for these speaker feeds, it is acceptible if the panning law does not produce a lower value of r_E at any other azimuth, and indeed we see from tables 1 and 2 that r_E is larger at all other azimuths. The requirement that r_E be broadly similar in value across most of the stereo stage means that the $\theta_V = \theta_E$ panpot law is in practice a good one, since the degradation of image stability in any position is in any case no worse than the inevitable image instability for azimuth $\frac{1}{2}\theta_3$.

It will be noted from tables 1 and 2, and from figs. 5 and 6, that the effect of increasing θ_3 from 30° to 45° is (i) to increase the angular width of images by 50%, as is expected, and (ii) to increase the values of $1-r_V$ and $1-r_E$ by a factor of about 2.25, i.e. the square of the factor by which angular width is increased, with a corresponding increase in image movement with change of listener position.

In audio-only applications, image movement is relatively unimportant provided that there is no geometric distortion within the image, i.e. provided the image shifts sideways as a whole. This is largely ensured if $1-r_E$ is almost constant, so that relatively wide reproduction stage widths such as $\theta_3 = 45^\circ$ can reasonably be used with the "optimal" 3-speaker panpot law. However, for TV use, where the image has to be locked into the apparent visual image position to within a few degrees, narrower layouts such as $\theta_3 = 30^\circ$ must be used to reduce image movement with listener position. However, even in this case, lack of geometric distortion of the image with listener movement ensures that the ears can clearly separate out different sound sources in the sound stage even if these do not precisely match the visual image for extreme listener positions.

4. OTHER 3-SPEAKER PANPOT LAWS

We now examine various non-optimised 3-speaker panpot laws that have been or might be proposed, in order to see what can go wrong.

First we look at the case where the C_3-speaker feed is always zero, i.e. the traditional 2-speaker stereo constant-power or sine/cosine panpot law (see for example Orban [17]). Such panpots ideally satisfy a law in which the respective left and right channel gains have the form

$$\cos(45^\circ - \theta) = \cos[(1-k)45^\circ] \tag{26a}$$

and

$$\cos(45^\circ + \theta) = \cos[(1+k)45^\circ], \tag{26b}$$

where $-1 \leq k \leq 1$ or $-45^\circ \leq \theta \leq 45^\circ$ are panpot parameters that determine position from right to left.

Figure 7 shows graphs of the 4 localisation parameters r_V, θ_V, r_E and θ_E for such conventionally panned 2-speaker stereo via a speaker layout subtending the standard 60° at the listener. It will be seen that the velocity and energy vector localisations agree only for centre and extreme left and right positions, and that elsewhere, the energy-vector localisation is wider than the velocity localisation, by a factor 2 near the centre of the stage. This discrepancy, which results in a wider image at frequencies above about 700 Hz than at lower frequencies, has long been known [18-20], and results in increased listening fatigue. However, due to the overlapping frequency ranges of the different localisation theories, these discrepancies cannot be rectified entirely by a frequency-dependent "shuffling" circuit, as was shown by Harwood [19] at the BBC.

It will be noted from fig. 7 that the energy vector azimuth θ_E is pulled into the location of the nearest speaker for sounds panned over to the left or right sides of the stage, even if not panned to the very extreme locations. This so-called "detent" effect was noted by Harwood [19] experimentally. From fig. 7, it will also be noted that the central images are quite unstable, with $1 - r_E = 0.1340$.

Possible alternative panpot laws for 3 speaker reproduction can be derived by passing a 2-channel panned signal with gains (26) into a 3×2 matrix circuit. In reference [13], figures 13 to 15, the 4 localisation parameters for such matrixed panned signals were plotted, and it was shown that for suitable matrix equations, a useful improvement in localisation quality and stability could be obtained.

Figure 8 shows the panpot law gains proposed in a 1934 paper [1] for 3-speaker stereo, plotted against the notional localisation in feet from a central location. Figure 9 shows the computed localisation parameters for this "Bell" law via a $\theta_3 = 45^\circ$ 3-speaker stereo layout.

It will be seen that : (i) the "Bell" panpot law gives wider energy vector localisation azimuths than velocity vector azimuths, (ii) the total stage width obtained does not cover the whole available stage, and (iii) that $1-r_E$ is particularly large, resulting in relatively poor image stability.

Attempts at a more systematic basis for designing panpot laws led the author, as reported in [7], to devise a 3-speaker panpot law that satisfies interaural phase localisation theories below 700 Hz for listeners in the stereo seat, perhaps the most obvious extension of Blumlein's derivation in 1931 of conventional 2-speaker panning* [8]. Because of the extra freedom of control of the sound field given by the use of three loudspeakers, it is possible to get these interaural phase cues correct for all head orientations of the listener, and this is achieved by ensuring that $r_V = 1$ for all localisation directions.

If such a panpot is normalised so that the total pressure gain equals one, and has a velocity vector localisation θ_V, then:

$$L_3 + C_3 + R_3 = 1 \tag{27a}$$

$$C_3 + (L_3 + R_3)\cos\theta_3 = \cos\theta_V \tag{27b}$$

and

$$(L_3 - R_3)\sin\theta_3 = \sin\theta_V \,, \tag{27c}$$

from which we get the panpot law:

$$L_3 = \tfrac{1}{2}\left[\frac{1-\cos\theta_V}{1-\cos\theta_3} + \frac{\sin\theta_V}{\sin\theta_3}\right] \tag{28a}$$

$$C_3 = \frac{\cos\theta_V - \cos\theta_3}{1 - \cos\theta_3} \tag{28b}$$

and

$$R_3 = \tfrac{1}{2}\left[\frac{1-\cos\theta_V}{1-\cos\theta_3} - \frac{\sin\theta_V}{\sin\theta_3}\right] . \tag{28c}$$

This "discrete" panpot law with ideal velocity vector localisation is shown for $\theta_3 = 45^\circ$ in figure 10, and the 4 localisation parameters at different panpot settings are shown in fig. 11. It will be seen that, while velocity vector localisation is perfect, by design, the energy vector localisation law is very bad indeed, with a very severe detent effect at the position of the centre speaker, with sounds at positions anywhere near the centre being pulled right into the centre speaker. This severe central detent effect rules out the panpot law of equs. (28) from serious use despite its excellent velocity vector performance. From fig. 11, it will also be seen that there is a (much less severe) detent effect at the left and right speaker positions. The law of equs. (28) provides an example of the

* It is worth noting that a note by Blumlein in EMI archives dated 1932 shows that Blumlein was well aware of the importance of a number of other auditory localisation mechanisms for stereo panning besides the interaural phase cue below 700 Hz.

danger of designing panpot laws for too narrow a range of psychoacoustic criteria - particularly only those relating to low-frequency interaural phase localisation cues.

Another "discrete" panpot law, widely used, is the "pairwise pan" law for three-speaker stereo, whereby sounds intended for localisation between the directions of two adjacent speakers are panned only to those two speakers, with other speaker gains equal to zero. Thus the panpot law is

$$L_3 = \sin 90k,\ C_3 = \cos 90k\ ,\ R_3 = 0 \tag{29a}$$

for $0 \leq k \leq 1$, and

$$L_3 = 0,\ C_3 = \cos 90k\ ,\ R_3 = -\sin 90k \tag{29b}$$

for $-1 \leq k \leq 0$, where $-1 \leq k \leq 1$ is a panpot setting parameter. Figure 12 shows the channel gains of this pairwise panpot law, and figure 13 shows the localisation parameters for this law at different panpot settings. While the velocity vector azimuth is well-behaved, it will be seen that there is marked detent effect for energy vector localisation at all three speaker locations, as well as a very non--uniform behaviour of r_V and r_E. This is another example of the poor localisation behaviour of a "discrete" panpot law.

5. 4-SPEAKER PANPOT LAWS

While the "optimal" 3-speaker panpot law is essentially uniquely defined by the requirement $\theta_V = \theta_E$, determining what is meant by an "optimal" 4-speaker panpot law is not so easy, because there is a one-parameter family of speaker feed gains for which $\theta_V = \theta_E = \theta$ even after total energy gain is normalised to one.

Different choices of speaker gains for any given choice of $\theta_V = \theta_E = \theta$ give different values of r_V and r_E, and making a choice that meets all the requirements (i) to (viii) above is not possible; one needs to make what can be regarded as an "arbitrary" choice or compromise between them. For example, should one go for a very high degree of constancy of r_E, should one attempt to make $r_V = 1$ (this proves to be possible only for some sound directions, and leads to a very non-constant r_E), should one go for a maximum possible value of r_E even if this is very non-constant, or should one go for a reasonably constant value of r_E but attempting to make sure that r_V is never allowed to go too small?

Figure 14 shows the speakers, and associated layout angles, of a 4-speaker stereo layout, where all speakers are supposed equidistant from the listener in the ideal stereo seat. We assume that $\theta_5 = \frac{1}{3}\theta_4$, so that the angle between adjacent pairs of speakers is the same value $\theta_3' = 2\theta_4 = \frac{2}{3}\theta_5$ for each adjacent pair.

We consider three different 4-speaker stereo panpot laws broadly satisfying $\theta_V = \theta_E$. The first such law is obtained simply by matrixing the optimal 3-speaker panpot law given above by passing it through a 4×3 "preservation" matrix decoder, as shown in figure 15 and described in ref. [13]. Such "presevation" decoder matrices have the property of substantially preserving the localisation parameters of 3-speaker stereo recordings when reproduced by 4 speakers. For $\theta_3 = 45^o$ and $\theta_4 = 3\theta_5 = 50^o$, it was shown in [13] that the 4×3 preservation decoder satisfies the matrix equation:-

$$\begin{pmatrix} L_4 \\ L_5 \\ R_5 \\ R_4 \end{pmatrix} = \begin{pmatrix} 0.9303 & -0.1287 & 0.0527 \\ 0.3314 & 0.6951 & -0.1479 \\ -0.1479 & 0.6951 & 0.3314 \\ 0.0527 & -0.1297 & 0.9303 \end{pmatrix} \begin{pmatrix} L_3 \\ C_3 \\ R_3 \end{pmatrix} \tag{30}$$

Table 4 tabulates the four speaker gains and the localisation parameters for the panpot of figure 15 for $\theta_3 = 45^o$ and $\theta_4 = 3\theta_5 = 50^o$; comparing this with table 2 shows that the localisation parameters are broadly similar, except that r_E peaks at around 0.97 near the edges of the stereo stage rather than at $r_E = 1$. This 4×3 panpot law only covers about 90^o of the available 100^o sound stage width.

A much better 4-speaker panpot law with $\theta_V = \theta_E$ is obtained if one uses the "optimal" 3-speaker panpot law for each half of the 4-speaker stereo stage, feeding sounds to the left of centre to the L_4, L_5 and R_5 speakers putting $R_4 = 0$, and feeding sounds to the right of centre to the L_5, R_5 and R_4 speakers putting $L_4 = 0$. This "piecewise 3-speaker optimal" 4-speaker panpot law is tabulated, for the left half of the stereo stage, in table 5, and its channel gains plotted in figure 16, and its localisation parameters plotted in figure 17. This 4-speaker panpot law covers the full stage width, has smooth and reasonably constant r_E over about 80% of that stage, and a value of $1-r_E$ and $1-r_V$ that is much smaller than that for the 4×3 panpot law, giving markedly better image stability. The main disadvantages of the piecewise 3-speaker optimal law are that :
(i) r_V does not vary in a smooth way near front centre, and
(ii) r_V falls to a rather smaller value than is desirable in the directions of the L_5 and R_5 speakers.

However, if one attempts to design a 4-speaker panpot law to achieve both $\theta_V = \theta_E$ and $r_V = 1$, one runs into other problems. Without going into full details, one can design such a panpot law as follows. Using the methods described in connection with equs. (27) and (28) above, for any chosen localisation azimuth $\theta = k\theta_4$, determine the speaker feed gains summing to one for the triple L_4, L_5 and R_5 of speakers having $\theta_V = \theta$ and $r_V = 1$, and in the same way compute speaker feed gains summing to one for the triple L_5, R_5 and R_4 of speakers having $\theta_V = \theta$ and $r_V = 1$. Then any linear combination of these two sets of speaker feed gains also has $\theta_V = \theta$ and $r_V = 1$; one adjusts the coefficients of the linear combination until one finds that linear combination (if any) whose computed value of θ_E equals θ_V.

Doing this, it is found that it is only possible to ensure that $\theta_V = \theta_E = \theta$ and $r_V = 1$ for $|\theta| \leq \theta_4$ and for two sectors near the L_5 and R_5 speakers. Table 6 tabulates the speaker feed gains and computed localisation parameters for this $\theta_V = \theta_E$ and $r_V = 1$ 4-speaker panpot law for

$\theta_4 = 3\theta_5 = 50^o$. It will be seen that r_E is very nonconstant even across each of the three sectors for which this law is defined, i.e. $|\theta| \leq 16\frac{2}{3}^o$ and $35.5^o \leq |\theta| \leq 50^o$. Thus sideways movement of the listener will cause quite severe geometric image distortions. Nevertheless, it is possible that this law with $\theta_V = \theta_E$ and $r_V = 1$ might have some applications to TV use with the inner sector covering the width of a TV screen and the outer sectors extreme "stage-off" sounds. However, the limited stage coverage and varying image stability makes this law unsuitable for general use.

One seeks a 4-speaker panpot law retaining $\theta_V = \theta_E$ with reasonable uniformity of r_E across most of the stage, but which, unlike the piecewise 3-speaker law of table 5, is smoothly continuous and which avoids excessively low values of r_V near azimuths $\pm\ \theta_5$. To some extent, such a law must compromise the uniformity of r_E near the azimuths of speakers L_5 and R_5 in order to increase r_V, and the choice has to be a nicely judged, but somewhat arbitrary, compromise.

While there are various intermediate 4-speaker panpot laws with $\theta_V = \theta_E$, it has been found that attempts to reduce the variations in r_V below those of the piecewise 3-speaker law of table 5 invariably cause an increased variation of r_E, so that, apart possibly from slightly modifying the law of table five near centre front in order to "smooth" the sharp corner in the r_V curve, there seems little to be gained from departing to any great degree from the piecewise 3-speaker law.

Because of the ill-defined nature of the compromises involved, it is much more difficult to identify which of the $\theta_V = \theta_E$ 4-speaker panpot laws is to be considered "optimal", unlike in the 3-speaker case where the choice is essentially unique. However, an approximation to the piecewise 3-speaker law is fairly close to a law giving a reasonably constant r_E, and the increase in image stability of this 4-speaker law over that given by a 3-speaker panpot law, including one matrixed into 4 speakers as described by equ. (30), is substantial, typically reducing image instability effects by a factor 2.

6. DOWNWARD COMPATIBILITY

One of the advantages of using an "optimal" panpot law is that generally this gives improved "downward compatibility" when 3 or 4 speaker stereo signals are matrixed to give mono or 2-speaker stereo, as compared to the use of pairwise panpots; in particular, the variations in reproduced total energy gain as a sound is panned across the stereo stage are considerably reduced across 95% of the stage.

By way of example, we consider the optimal 3-channel panpot fed into the 3-channel transmission system described in ref. [14]. That system uses three transmission channel signals M, S and T given by

$$\begin{pmatrix} M \\ S \\ T \end{pmatrix} = \begin{pmatrix} 0.5000 & 0.7071 & 0.5000 \\ 0.7071 & 0.0000 & -0.7071 \\ 0.5000 & -0.7071 & 0.5000 \end{pmatrix} \begin{pmatrix} L_3 \\ C_3 \\ R_3 \end{pmatrix}, \qquad (31)$$

with the inverse decoding equations

$$\begin{pmatrix} L_3 \\ C_3 \\ R_3 \end{pmatrix} = \begin{pmatrix} 0.5000 & 0.7071 & 0.5000 \\ 0.7071 & 0.0000 & -0.7071 \\ 0.5000 & -0.7071 & 0.5000 \end{pmatrix} \begin{pmatrix} M \\ S \\ T \end{pmatrix}. \quad (32)$$

Mono reception derives simply the signal M, and 2-channel stereo reception derives respective left and right signals

$$\begin{pmatrix} L \\ R \end{pmatrix} = \begin{pmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{pmatrix} \begin{pmatrix} M \\ S \end{pmatrix}. \quad (33)$$

Table 7 computes, for the optimal 3-speaker panpot for a 3-speaker layout with $\theta_3 = 45^{\circ}$, as tabulated in table 2, the actual gains of the transmitted M, S and T signals, and the reproduced gains in dB of mono and 2-channel stereo reception of these signals. The reproduced 2-channel stereo position of signals is also computed as the angle (in degrees)

$$\tan^{-1}[(L-R)/(L+R)] \quad (34)$$

which runs from -45° at due right through 0° at centre to $+45^{\circ}$ at due left, with values greater than 45° indicating antiphase signals beyond the left speaker.

It will be seen from table 7 that the stereo gain compatibility of the 3-speaker optimal panpot is remarkably good, with a gain variation of only 0.344 dB across the central 94% of the stage; only at the outmost extremes does the gain drop by 0.905 dB relative to centre front and a total gain variation of 1.249 dB. The stereo positioning in 2-channel reception is also reasonable, extending a little outside the two stereo speakers. The mono compatibility is not so good, with a gain variation of 5.68 dB, most of this occuring at the outmost extremes of the 3-channel stereo stage, with only 3 dB variation across the central 82% of the stage. It will be noted that, as with conventional 2-speaker stereo panpots, the mono gain is highest for central sounds, falling away smoothly to either side.

The situation using pairwise panpots for 3-channel stereo is considerably worse in 2-channel stereo reception, with central images reproduced 3.01 dB down, rising to a level of 0 dB gain for sounds panned to just beyond half left, and falling again to - 1.25 dB at the edge of the stage, with 2-speaker <u>positioning</u> similar to the optimal panpot. The mono gain for centre sounds is - 3.01 dB, rising to -1.25 dB for sounds about 40% to the left, then falling to - 6.02 dB at the extreme stage edge. While the total gain variation in mono of the pairwise panpot is less (at 4.77 dB) than the 5.68 dB of the "optimal" panpot, the gain variation near the centre of the stage is larger, with central reproduced 1.63 dB <u>quieter</u> than half-left or half-right sounds.

The mono compatibility of the 3-channel transmission system equ. (31) is identical to that of other systems proposed in Meares [5] and Theile [6]; the stereo reproduction of our proposal, however, has increased width (difference gain) via two speakers of 3.01 dB, which improves the gain uniformity, especially with the optimal 3-speaker panpot.

Our conclusion is that the use of the optimal 3-speaker panpot considerably improves 2-channel stereo compatibility as compared to the use of a 3-speaker pairwise panpot, and only marginally worsens maximum gain variations in mono, but gives a much better pattern of mono gain variations across the central parts of the stereo stage.

The use of the 4-channel panpot of table 5 can be similarly shown to produce less overall gain variation when reduced to both 2- and 3-speaker stereo via the reception systems described in ref. [14] than does pairwise panning, but we postpone a detailed analysis of this case to a future paper. The mono compatibilty remains quite reasonable, falling away smoothly from the centre of the stage, but the extreme edges of the 4-speaker stage are reproduced in mono 6.29 dB quieter than sounds at the centre - this also being true for pairwise panned 4-speaker sounds, but with a greater degree of local gain variation, being 3.01 dB down at ⅓-left, going back up to 1.34 dB down around ½-left, then falling to 6.29 dB down at hard left.

We conclude that optimal panpots do affect the compatibility via hierarchical transmission systems, but that they generally give better results than pairwise panpots in stereo reception modes, and generally comparable results in mono reception modes, but with a much smoother gain variation law whose behaviour is easier to understand.

7. CONCLUSIONS

This paper has used a simple theory of auditory localisation that takes into account both interaural phase and amplitude cues to design panpots for 3- and 4-speaker stereo that give consistent localisation according to several cues, at least in the conditions of a domestic listening environment. The resulting "optimal" panpot laws do not vary greatly with the angle subtended by the speaker layout at the listener, and have good compatibility when reduced to mono or two channel stereo.

We have given detailed analyses of the psychoacoustic performance of several previously proposed 3-speaker panpot laws, including an historic law of Bell Telephone Laboratories, a pairwise panpot law, and a law optimised only for interaural phase cues, and shown that all have quite or very poor image quality and stability performance.

In particular, the 3-speaker "optimal" panpot law was shown to be essentially uniquely defined, although there is a minor degree of choice in laws for 4-speaker stereo. These laws are recommended as a basis for production work in multispeaker stereo, although for some purposes (e.g. hard-centre images), other laws may be used. In a future publication, we shall describe practical algorithms for approximating these panpot laws in mixing desks.

REFERENCES

[1] J.C. Steinberg & W,B, Snow, "Auditory Perspective - Physical Factors," Electrical Engineering, vol. 53 no. 1, pp. 12-15 (1934 Jan.), reprinted in ref. [16]

[2] W.B. Snow, "Basic Principles of Stereophonic Sound," J. SMPTE vol. 61, pp. 567-589 (1953 Nov.). Reprinted in ref [16].

[3] D.J. Meares, "Sound Systems for High Definition TV," Proc. Inst. Acoustics, vol. 11 part 7, pp. 47-62 (1989 Nov.)

[4] S. Komiyama, "Subjective Evaluation of Angular Displacement Between Picture and Sound Directions for HDTV Sound Systems," J. Audio Eng. Soc., vol. 37 no. 4, pp. 210-214 (1989 April)

[5] D.J. Meares, "High-Quality Sound for High Definition Television," Proc. 10th International AES Conference "Images of Audio" London, England, Sept. 1991 pp. 163-172

[6] G. Theile, "HDTV Sound Systems: How Many Channels?" Proc. 10th AES International Conference "Images of Audio", London, England, Sept. 1991 pp. 147-162

[7] M.A. Gerzon, "Three Channels - The Future of Stereo?," Studio Sound, vol. 32 no. 4, pp.112-125 (1990 June)

[8] A.D. Blumlein, U.K. Patent 394,325 filed 1931 Dec. 14, reprinted in part in ref. [16]

[9] D.H. Cooper, T. Shiga & T. Takagi, "QMX Carrier-Channel Disc," J. Audio Eng. Soc., vol. 21 no. 10 pp. 614-624 (1973 Oct.)

[10] M.A. Gerzon, "Surround Sound Psychoacoustics," Wireless World, vol. 80, pp. 483-486 (1974 Dec.)

[11] M.A. Gerzon, "General Metatheory of Auditory Localisation", to be presented at the 92nd Audio Eng. Soc. Convention, Vienna, Austria (1992 March). Originally written in 1976/7 under the title "The Rational Systematic Design of Surround Sound Recording and Reproduction Systems. Part I. General Theory of Directional Psychoacoustics and Applications", and submitted by NRDC to the FCC and EBU in proceedings on quadraphonic broadcasting.

[12] M.A. Gerzon, "The Optimum Choice of Surround Sound Encoding Specification," Preprint 1199, Audio Engineering Society 56th Convention, Paris 1977 March

[13] M.A. Gerzon, "Optimal Reproduction Matrices for Multispeaker Stereo," Preprint 3180 of the 91st Audio Engineering Society Convention, New York, Oct. 1991

[14] M.A. Gerzon, "Hierarchical Transmission System for Multispeaker Stereo," Preprint 3199 of the 91st Audio Engineering Society Convention, New York, Oct. 1991

[15] Y. Makita, "On the Directional Localization of Sound in the Stereophonic Sound Field," E.B.U. Review, Part A no. 73, pp. 102-108 (1962)

[16] ed. J. Eargle, Stereophonic Techniques, Audio Engineering Society, New York, 1986

[17] R. Orban, "Notes on Panpots," J. Audio Eng. Soc., vol. 19 no. 11, pp. 936-9 (1971 Nov.)

[18] H.A.M. Clark, G.F. Dutton & P.B. Vanderlyn, "The 'Stereosonic' Recording and Reproducing System," J. Audio Eng. Soc., vol. 6 no. 2 pp. 102-117 (1958 April). Reprinted in ref. [16]

[19] H.D. Harwood, "Stereophonic Image Sharpness," Wireless World, vol. 74, pp. 207-211 (1968)

TABLES

	Panpot gains			localisation parameters			
k	L_3	C_3	R_3	r_V	θ_V	r_E	θ_E
0.00	0.3326	0.8824	0.3326	0.9424	0.00	0.9704	0.00
0.10	0.4095	0.8753	0.2573	0.9434	3.00	0.9700	3.00
0.20	0.4866	0.8539	0.1848	0.9462	6.00	0.9690	6.00
0.30	0.5625	0.8145	0.1169	0.9509	9.00	0.9677	9.00
0.40	0.6363	0.7695	0.0548	0.9575	12.00	0.9665	12.00
0.50	0.7071	0.7071	0.0000	0.9659	15.00	0.9659	15.00
0.60	0.7743	0.6311	-0.0458	0.9760	18.00	0.9667	18.00
0.70	0.8375	0.5404	-0.0806	0.9874	21.00	0.9700	21.00
0.75	0.8675	0.4886	-0.0930	0.9935	22.50	0.9720	22.50
0.80	0.8964	0.4315	-0.1011	0.9996	24.00	0.9753	24.00
0.85	0.9242	0.3674	-0.1039	1.0054	25.50	0.9795	25.50
0.90	0.9509	0.2932	-0.0994	1.0105	27.00	0.9849	27.00
0.92	0.9612	0.2592	-0.0946	1.0120	27.60	0.9874	27.60
0.94	0.9713	0.2214	-0.0872	1.0131	28.20	0.9901	28.20
0.96	0.9811	0.1777	-0.0761	1.0134	28.80	0.9931	28.80
0.98	0.9908	0.1227	-0.0579	1.0119	29.40	0.9964	29.40
0.99	0.9954	0.0852	-0.0429	1.0097	29.70	0.9981	29.70
1.00	1.0000	0.0000	0.0000	1.0000	30.00	1.0000	30.00

Table 1. "Optimal" 3-speaker panpot law gains and computed localisation parameters for a 3-speaker layout with $\theta_3 = 30^o$, tabulated against parameter k that is the proportion of the half-stage angular width.

TABLES cont.

	Panpot gains			localisation parameters			
k	L_3	C_3	R_3	r_V	θ_V	r_E	θ_E
0.00	0.3426	0.8748	0.3426	0.8714	0.00	0.9313	0.00
0.10	0.4182	0.8681	0.2676	0.8734	4.50	0.9307	4.50
0.20	0.4933	0.8478	0.1945	0.8796	9.00	0.9291	9.00
0.30	0.5670	0.8142	0.1245	0.8901	13.50	0.9269	13.50
0.40	0.6385	0.7673	0.0592	0.9048	18.00	0.9248	18.00
0.50	0.7071	0.7071	0.0000	0.9239	22.50	0.9239	22.50
0.60	0.7725	0.6330	-0.0512	0.9472	27.00	0.9253	27.00
0.70	0.8344	0.5435	-0.0917	0.9745	31.50	0.9308	31.50
0.75	0.8640	0.4920	-0.1068	0.9892	33.75	0.9357	33.75
0.80	0.8929	0.4348	-0.1173	1.0041	36.00	0.9425	36.00
0.85	0.9209	0.3704	-0.1219	1.0185	38.25	0.9516	38.25
0.90	0.9481	0.2954	-0.1178	1.0310	40.50	0.9635	40.50
0.92	0.9587	0.2610	-0.1126	1.0347	41.40	0.9693	41.40
0.94	0.9693	0.2227	-0.1043	1.0371	42.30	0.9757	42.30
0.96	0.9797	0.1784	-0.0914	1.0372	43.20	0.9828	43.20
0.98	0.9900	0.1228	-0.0699	1.0327	44.10	0.9908	44.10
0.99	0.9950	0.0851	-0.0519	1.0263	44.55	0.9952	44.55
1.00	1.0000	0.0000	0.0000	1.0000	45.00	1.0000	45.00

Table 2. "Optimal" 3-speaker panpot law gains and computed localisation parameters for a 3-speaker layout with $\theta_3 = 45^o$, tabulated against parameter k that is the proportion of the half-stage angular width.

θ_3	0	15	30	45	60	75	90
dB	8.73	8.67	8.47	8.14	7.66	6.97	6.02

Table 3. Centre-to-edge crosstalk in dB for centre sounds of an "optimal" 3-speaker panpot for different layout angles θ_3.

	channel amplitude gains				localisation parameters			
k	L_4	L_5	R_5	R_4	r_V	θ_V	r_E	θ_E
0.00	0.2233	0.6709	0.6709	0.2233	0.8793	0.00	0.9266	0.00
0.10	0.2905	0.7024	0.6302	0.1584	0.8812	4.45	0.9264	4.52
0.20	0.3592	0.7240	0.5808	0.0970	0.8870	8.91	0.9258	9.03
0.30	0.4285	0.7354	0.5234	0.0401	0.8968	13.37	0.9252	13.50
0.40	0.4976	0.7362	0.4586	-0.0108	0.9105	17.84	0.9248	17.93
0.50	0.5661	0.7258	0.3869	-0.0545	0.9282	22.32	0.9254	22.32
0.60	0.6339	0.7035	0.3088	-0.0890	0.9498	26.81	0.9275	26.67
0.70	0.7009	0.6679	0.2240	-0.1119	0.9745	31.31	0.9324	31.01
0.75	0.7344	0.6441	0.1788	-0.1176	0.9877	33.57	0.9363	33.18
0.80	0.7681	0.6154	0.1313	-0.1185	1.0008	35.83	0.9413	35.37
0.85	0.8023	0.5806	0.0808	-0.1129	1.0130	38.10	0.9478	37.58
0.90	0.8375	0.5369	0.0260	-0.0980	1.0228	40.39	0.9559	39.85
0.92	0.8522	0.5158	0.0023	-0.0881	1.0252	41.30	0.9595	40.77
0.94	0.8674	0.4914	-0.0232	-0.0749	1.0261	42.22	0.9634	41.72
0.96	0.8835	0.4622	-0.0512	-0.0566	1.0245	43.15	0.9674	42.70
0.98	0.9014	0.4238	-0.0842	-0.0288	1.0178	44.09	0.9710	43.73
0.99	0.9119	0.3966	-0.1052	-0.0069	1.0099	44.56	0.9722	44.30
1.00	0.9303	0.3314	-0.1479	0.0527	0.9805	45.08	0.9690	45.08

Table 4. 4-speaker panpot law gains and localisation parameters obtained from the optimal 3-speaker panpot law for $\theta_3 = 45^\circ$ (see table 2) when reproduced via a 4×3 preservation matrix decoder. Note that θ_V and θ_E are almost equal, never differing by more than 0.55°. Compare the localisation parameters with those in table 2.

TABLES cont.

	channel amplitude gains				localisation parameters			
k	L_4	L_5	R_5	R_4	r_V	θ_V	r_E	θ_E
0.00	0.0000	0.7071	0.7071	0.0000	0.9580	0.00	0.9580	0.00
0.10	0.0861	0.7951	0.6004	0.0000	0.9433	5.00	0.9593	5.00
0.20	0.1866	0.8528	0.4878	0.0000	0.9334	10.00	0.9616	10.00
0.30	0.2965	0.8793	0.3727	0.0000	0.9293	15.00	0.9631	15.00
0.40	0.4111	0.8740	0.2591	0.0000	0.9302	20.00	0.9628	20.00
0.50	0.5258	0.8370	0.1518	0.0000	0.9363	25.00	0.9609	25.00
0.60	0.6367	0.7691	0.0555	0.0000	0.9476	30.00	0.9586	30.00
0.70	0.7410	0.6710	-0.0246	0.0000	0.9640	35.00	0.9582	35.00
0.75	0.7901	0.6104	-0.0568	0.0000	0.9739	37.50	0.9595	37.50
0.80	0.8369	0.5410	-0.0825	0.0000	0.9847	40.00	0.9623	40.00
0.85	0.8815	0.4615	-0.1002	0.0000	0.9961	42.50	0.9672	42.50
0.90	0.9236	0.3680	-0.1070	0.0000	1.0073	45.00	0.9745	45.00
0.92	0.9398	0.3250	-0.1054	0.0000	1.0113	46.00	0.9783	46.00
0.94	0.9556	0.2771	-0.1003	0.0000	1.0146	47.00	0.9826	47.00
0.96	0.9709	0.2217	-0.0901	0.0000	1.0169	48.00	0.9876	48.00
0.98	0.9858	0.1524	-0.0706	0.0000	1.0166	49.00	0.9933	49.00
0.99	0.9930	0.1055	-0.0530	0.0000	1.0141	49.50	0.9965	49.50
1.00	1.0000	0.0000	0.0000	0.0000	1.0000	50.00	1.0000	50.00

Table 5. Channel amplitude gains and psychoacoustic localisation parameters for 4-speaker panpot law based on piecewise 3-speaker optimal panpot . $\theta_4 = 3\theta_5 = 50^{\circ}$ case, tabulated against panpot setting parameter k that is the proportion of the half-stage angular width. Right half is mirror-symmetric to the left half stage tabulated in the table.

k	channel amplitude gains L_4	L_5	R_5	R_4	localisation parameters r_V	θ_V	r_E	θ_E
0.00	-0.0826	0.7023	0.7023	-0.0826	1	0.00	0.9573	0.00
0.10	-0.0469	0.8064	0.5804	-0.1030	1	5.00	0.9576	5.00
0.20	-0.0002	0.8958	0.4321	-0.1040	1	10.00	0.9693	10.00
0.25	0.0237	0.9356	0.3395	-0.0936	1	12.50	0.9782	12.50
0.30	0.0392	0.9735	0.2147	-0.0684	1	15.00	0.9898	15.00
⅓	0.0000	1.0000	0.0000	0.0000	1	16.67	1.0000	16.67
0.71	0.7893	0.5597	0.2060	-0.1462	1	35.50	0.9273	35.50
0.80	0.8355	0.5468	-0.0366	-0.0409	1	40.00	0.9627	40.00
0.90	0.9249	0.3606	-0.1197	0.0149	1	45.00	0.9734	45.00
0.95	0.9653	0.2321	-0.1166	0.0284	1	47.50	0.9829	47.50
1.00	1.0000	0.0000	0.0000	0.0000	1	50.00	1.0000	50.00

Table 6. 4-speaker stereo panpot law for $\theta_4 = 3\theta_5 = 50^o$ speaker layout for which $\theta_V = \theta_E$ and $r_V = 1$. The panpot setting k is the proportion of the half-stage width 50^o. The law is defined only for three angular sectors: $-\frac{1}{3} \leq k \leq \frac{1}{3}$ and $0.71 \leq |k| \leq 1$. Left half of stage shown; right half is mirror-symmetric.

TABLES cont

k	M	S	T	mono dB	stereo dB	stereo angle
0.00	0.9611	0.0000	-0.2760	-0.344	-0.344	0.00
0.10	0.9567	0.1065	-0.2709	-0.385	-0.331	6.35
0.20	0.9434	0.2113	-0.2556	-0.506	-0.293	12.63
0.30	0.9215	0.3129	-0.2300	-0.710	-0.236	18.75
0.40	0.8914	0.4096	-0.1938	-0.998	-0.166	24.68
0.50	0.8536	0.5000	-0.1464	-1.375	-0.094	30.36
0.60	0.8082	0.5824	-0.0869	-1.849	-0.033	35.78
0.70	0.7556	0.6549	-0.0130	-2.434	-0.001	40.91
0.75	0.7265	0.6865	0.0308	-2.775	-0.004	43.38
0.80	0.6952	0.7143	0.0803	-3.158	-0.028	45.78
0.85	0.6614	0.7373	0.1376	-3.591	-0.083	48.11
0.90	0.6240	0.7537	0.2062	-4.096	-0.189	50.38
0.92	0.6076	0.7575	0.2385	-4.327	-0.254	51.27
0.94	0.5900	0.7592	0.2750	-4.584	-0.341	52.15
0.96	0.5703	0.7574	0.3180	-4.878	-0.463	53.02
0.98	0.5469	0.7495	0.3731	-5.242	-0.651	53.88
0.99	0.5317	0.7403	0.4114	-5.486	-0.805	54.31
1.00	0.5000	0.7071	0.5000	-6.021	-1.249	54.74

Table 7. Mono and stereo compatibility of "optimal" 3-speaker panpot law for layout with $\theta_3 = 45^o$ via the 3-channel encoding system of equ. (31), showing gains in dB for mono and 2-channel stereo reception, reproduced 2-channel stereo angle $\tan^{-1}[(L-R)/(L+R)]$, and gains of M, S and T transmission signals.

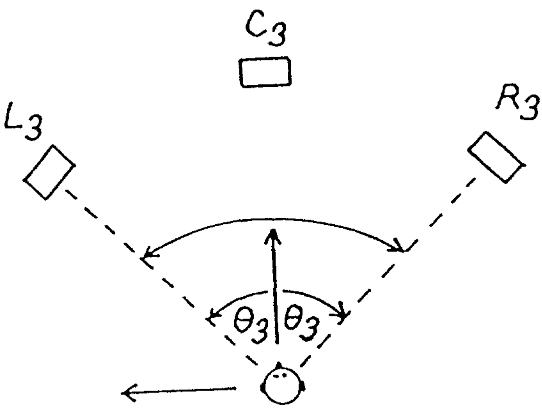

Figure 1. 3-speaker stereo layout with all speakers at the same distance from a central listener.

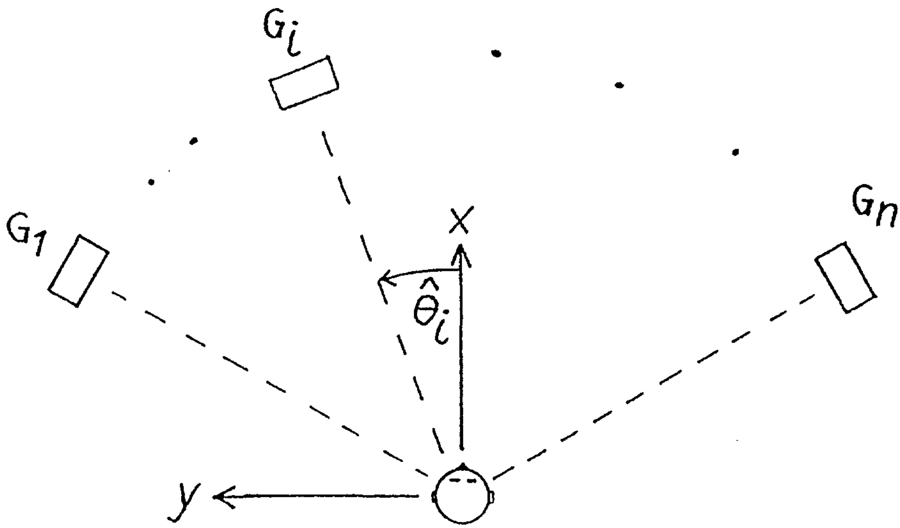

Figure 2. Showing the speaker-feed gains G_i of n speaker panpot reproduction from speakers equidistant from an ideal centre position, showing x- and y- axes and speaker direction angles.

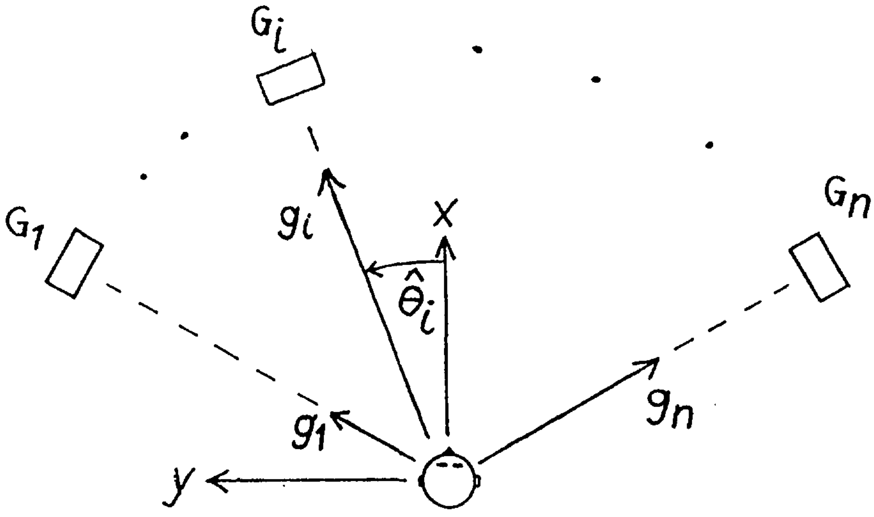

Figure 3. Showing the vectors pointing to n speakers at the central position, each having length equal to the "magnitude" g_i of sound gain from the i'th speaker. The localisation vector equals the vector sum of these vectors divided by the scalar sum of the g_i's.

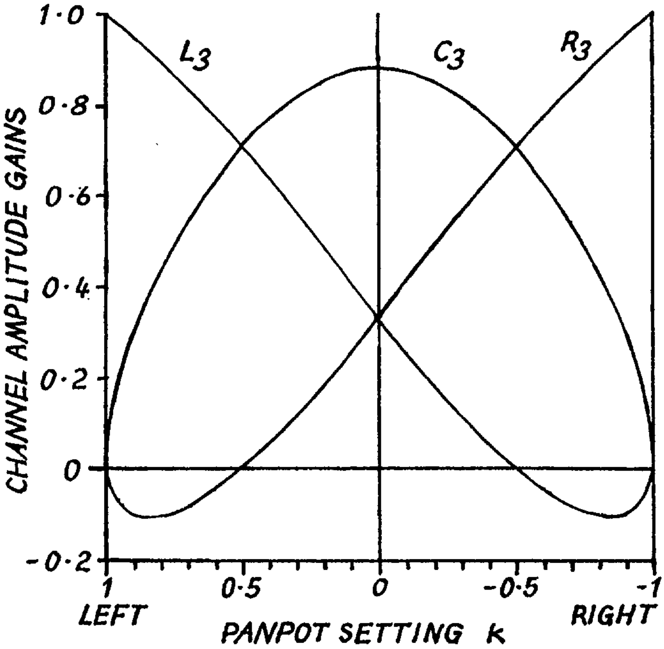

Figure 4. "optimum" 3-speaker stereo panpot law, having $\theta_V = \theta_E$, for $\theta_3 = 30^o$ speaker layout. The gain law for $\theta_3 = 45^o$ looks very similar.

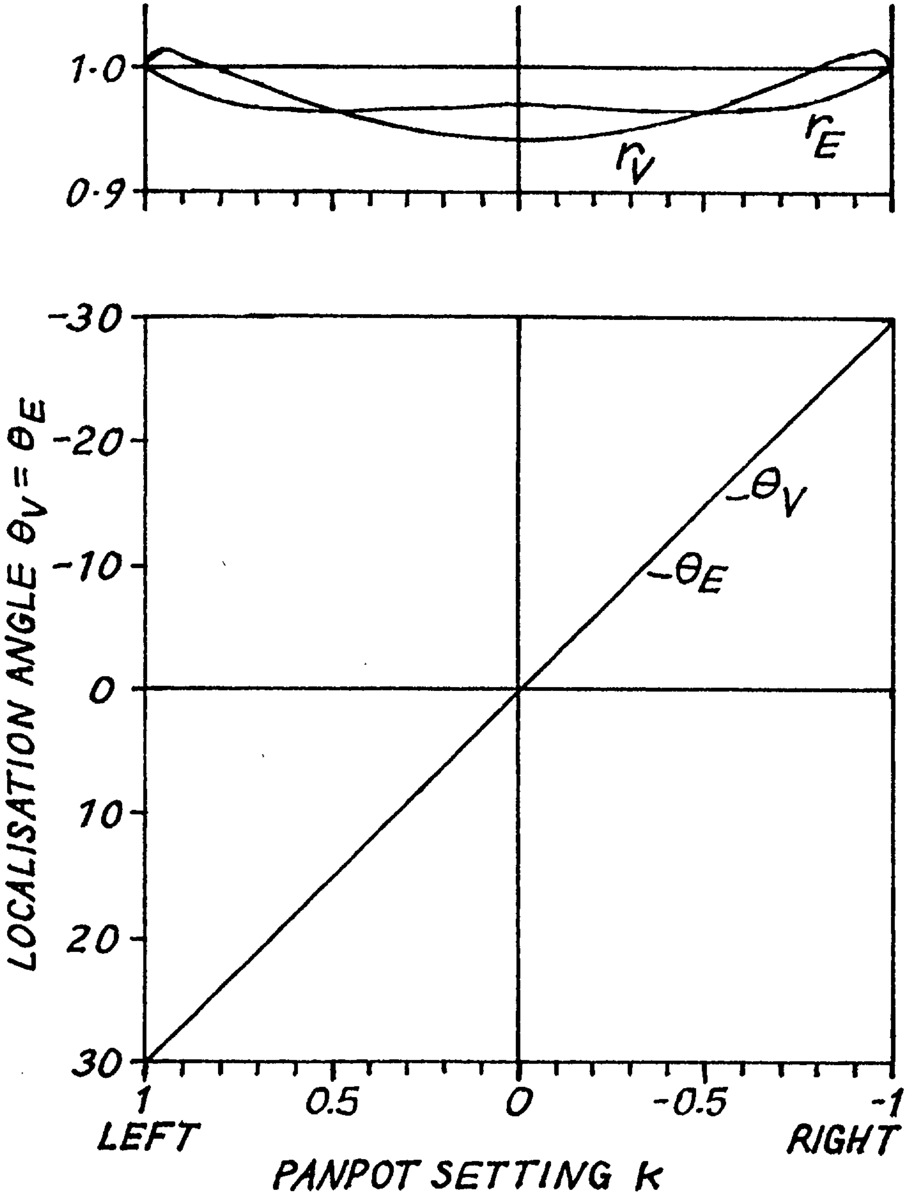

Figure 5. Localisation parameters r_V, Θ_V, r_E and Θ_E for the "optimal" 3-speaker panpot law of table 1 and figure 4, for speaker layout with $\Theta_3 = 30^o$.

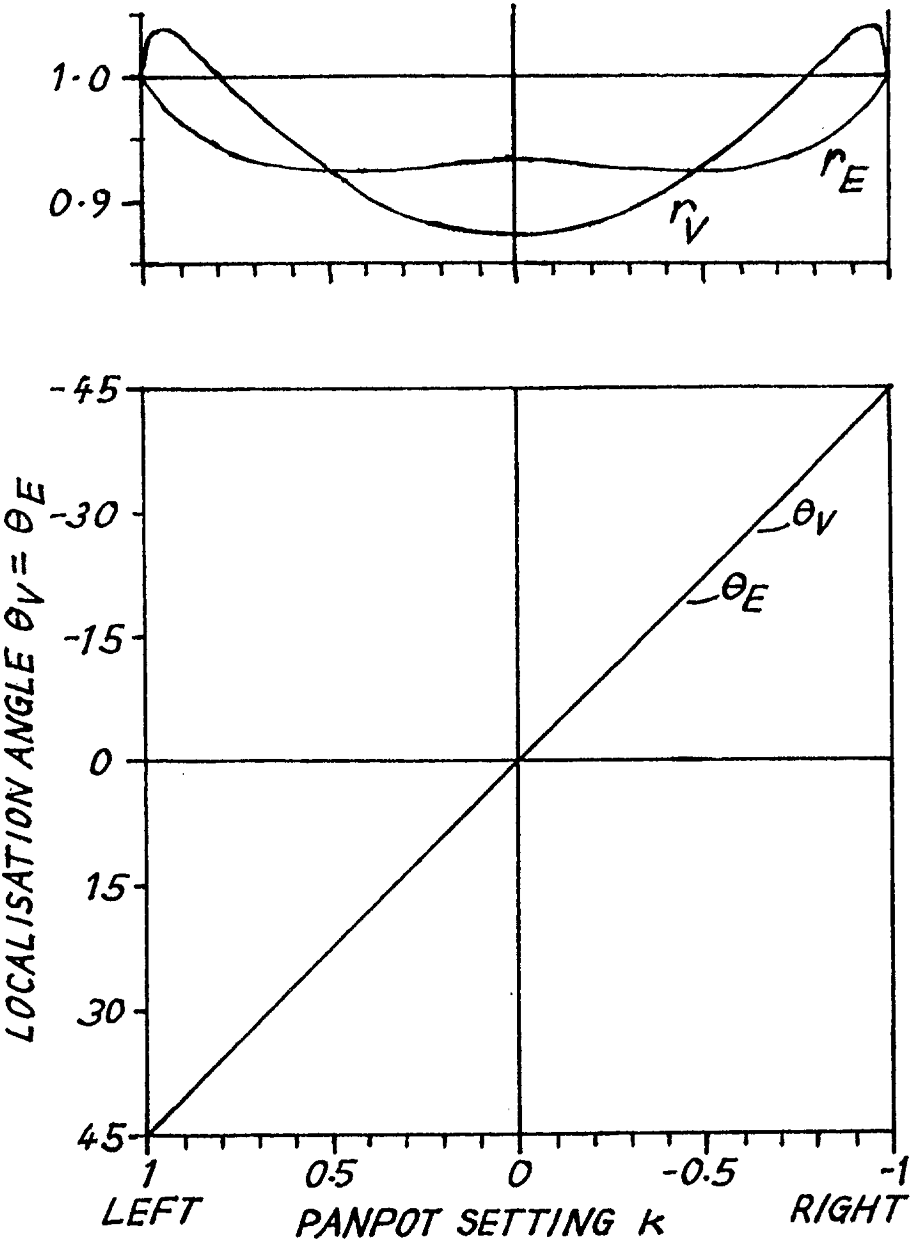

Figure 6. Localisation parameters r_V, θ_V, r_E and θ_E for the "optimal" 3-speaker panpot law of table 2 for a speaker layout with $\theta_3 = 45^o$.

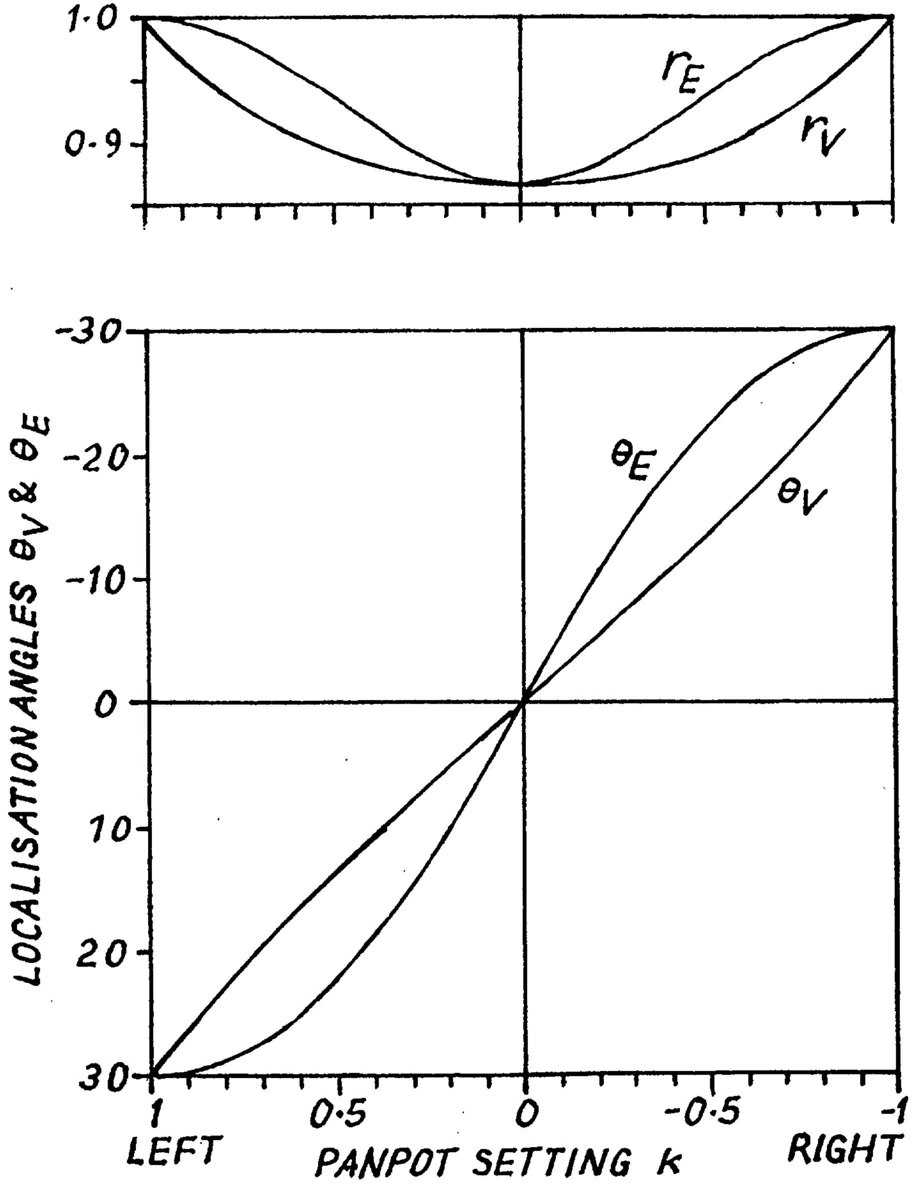

Figure 7. Localisation parameters r_V, θ_V, r_E and θ_E for 2-speaker stereo reproduction of a sine/cosine panpot via a standard 2-speaker stereo layout subtending 60°.

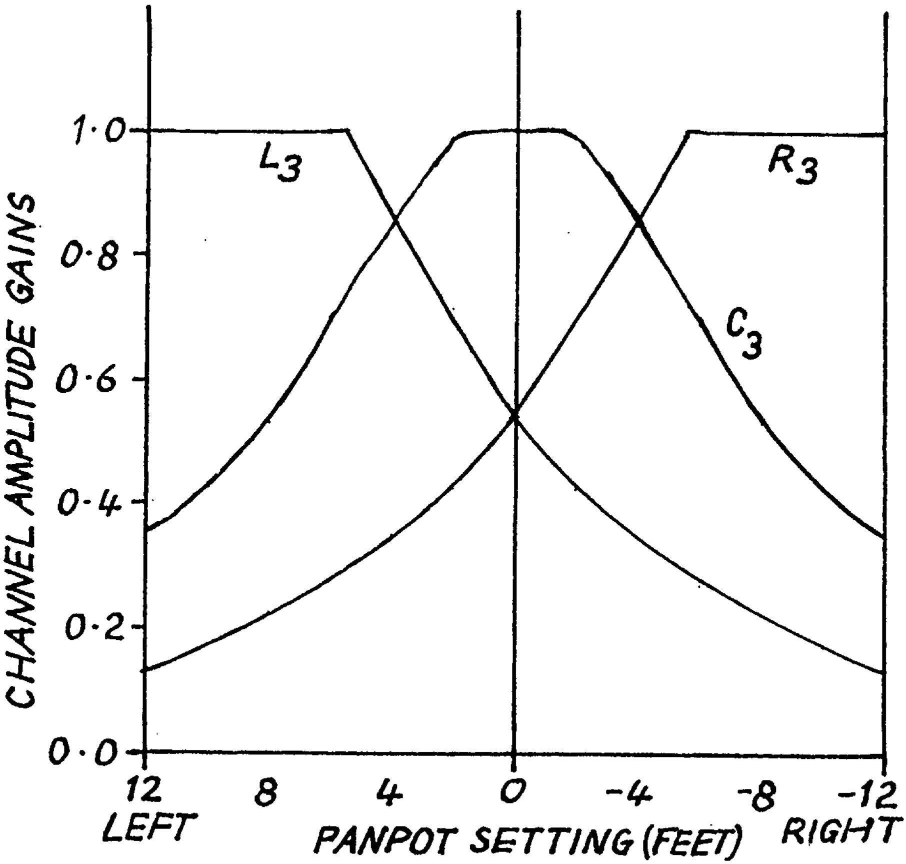

Figure 8. Channel gains of the 1934 Bell Telephone Laboratories 3-speaker panpot [1], plotted against nominal localisation in feet.

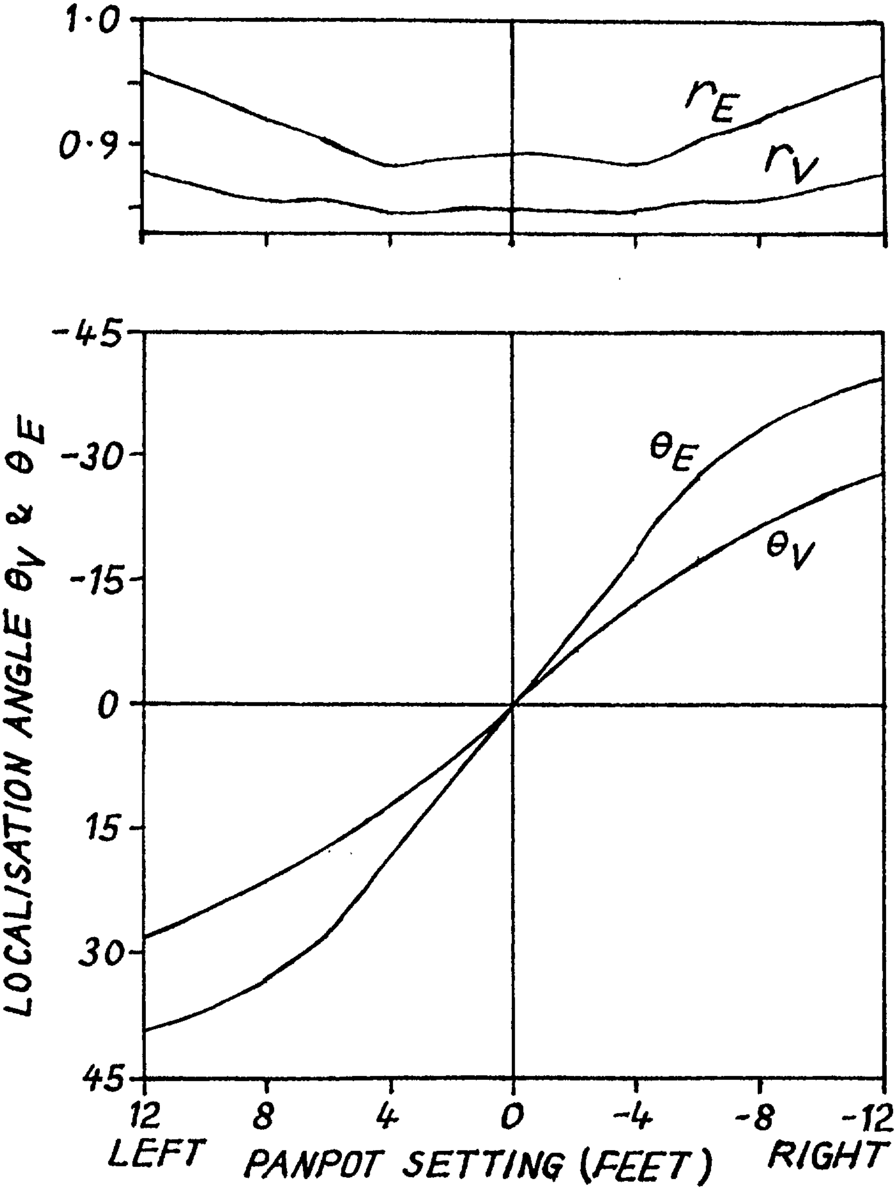

Figure 9. Localisation parameters r_V, Θ_V, r_E and Θ_E for 1934 Bell 3-speaker panpot law of figure 8 reproduced via equidistant speaker layout with $\Theta_3 = 45^o$.

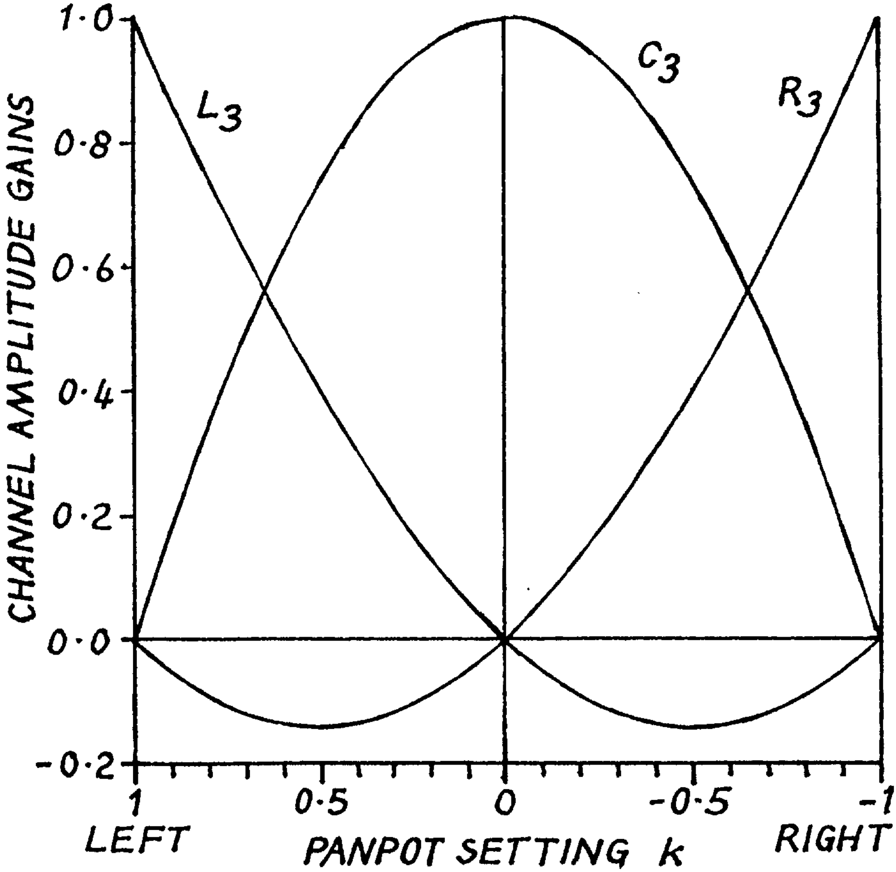

Figure 10. 3-speaker panpot law gains giving ideal interaural phase localisation for all head orientations for a central listener for $\theta_3 = 45^{\circ}$. The law for other values of θ_3 between 0° and 80° is similar.

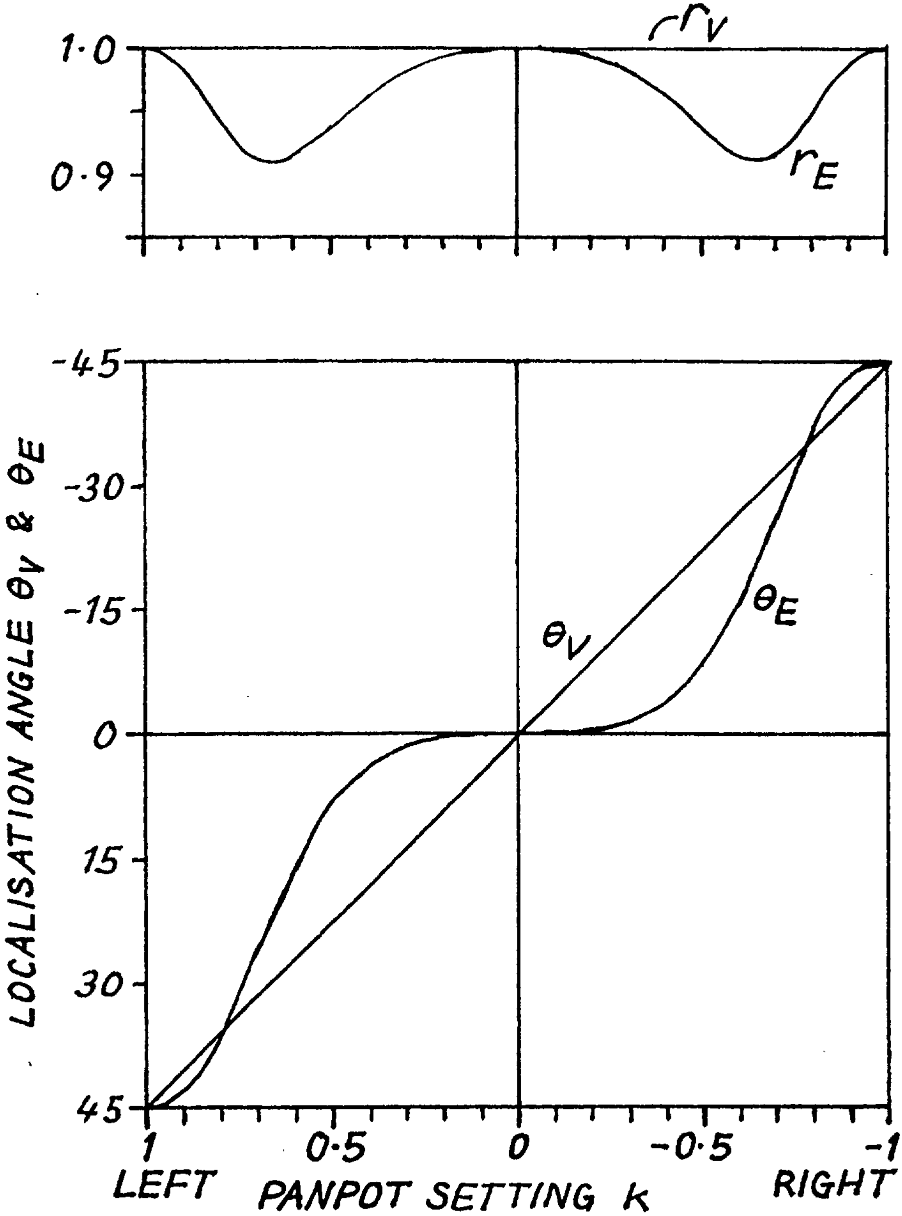

Figure 11. Localisation parameters r_V, Θ_V, r_E and Θ_E for "ideal" interaural-phase 3-speaker panpot law of figure 10 via layout with $\Theta_3 = 45^o$.

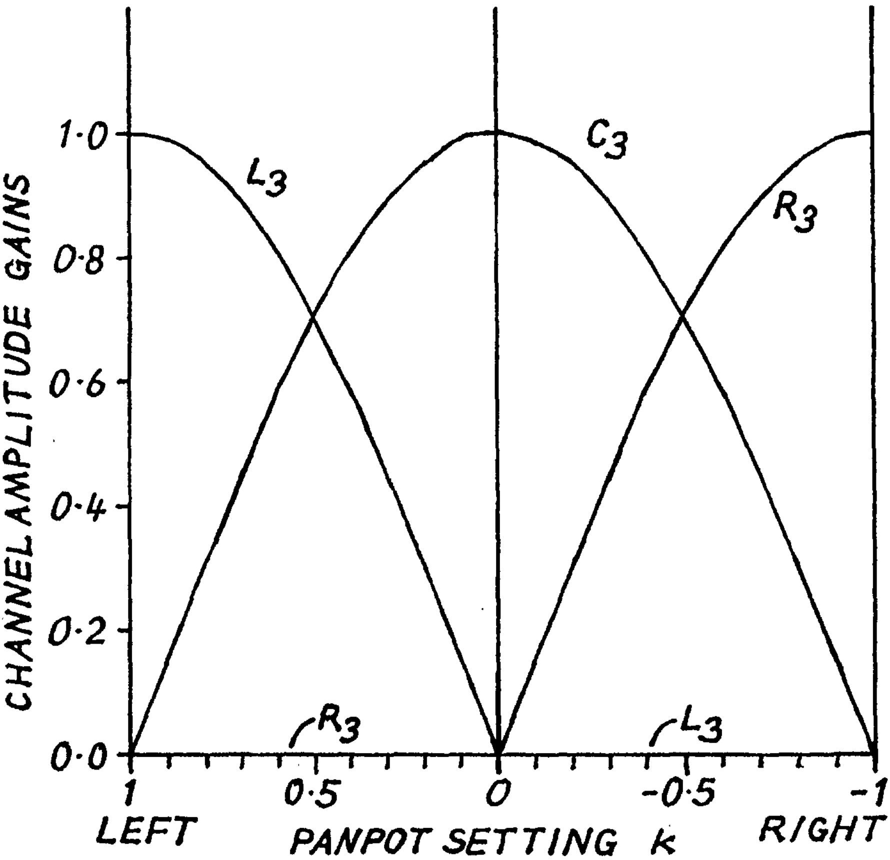

Figure 12. Constant-power pairwise panpot law for a 3-speaker stereo layout.

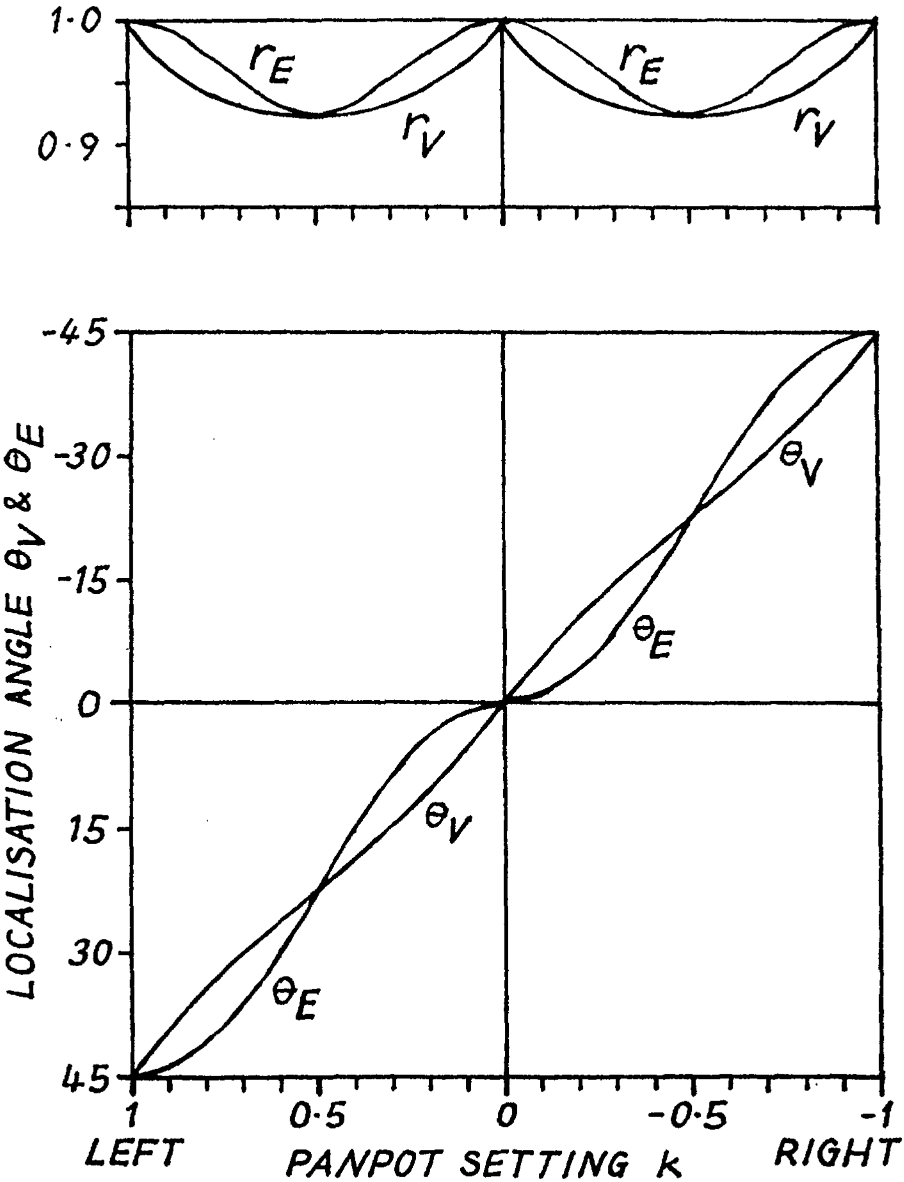

Figure 13. Localisation parameters r_V, Θ_V, r_E and Θ_E for pairwise panpot law of figure 12 via 3-speaker stereo layout with $\Theta_3 = 45^o$.

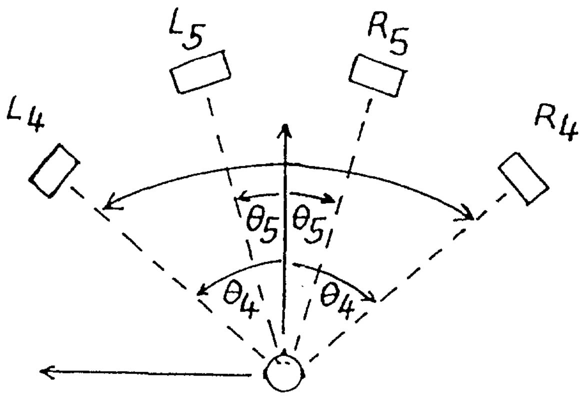

Figure 14. Showing 4-speaker stereo layout with all speaker equidistant from a central listener, showing angles and speaker notations.

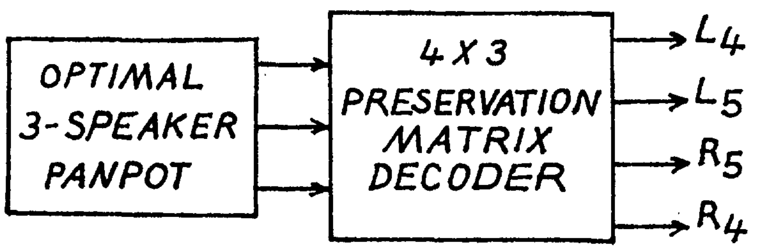

Figure 15. 4-speaker stereo panpot derived from a 3-speaker optimal panpot via a 4 ×3 preservation matrix decoder.

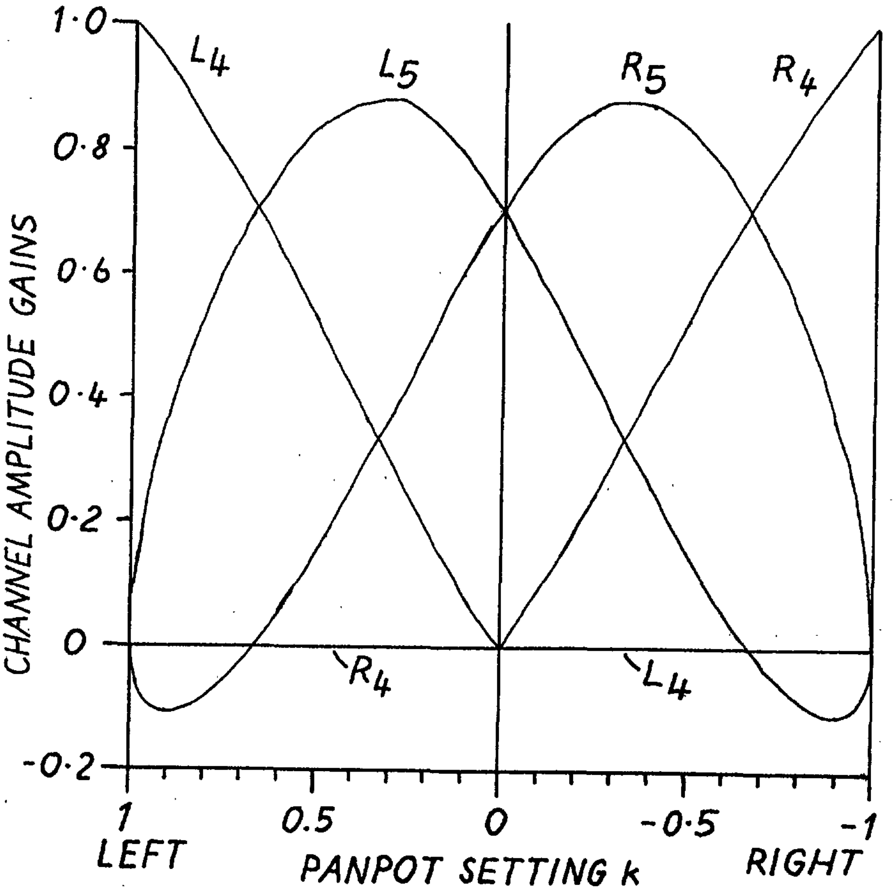

Figure 16. Gains of channels L_4, L_5, R_5, R_4 of 4-speaker panpot law based on piecewise 3-channel optimal panpot of table 5 for 4-speaker layout with $\theta_4 = 3\theta_5 = 50^o$.

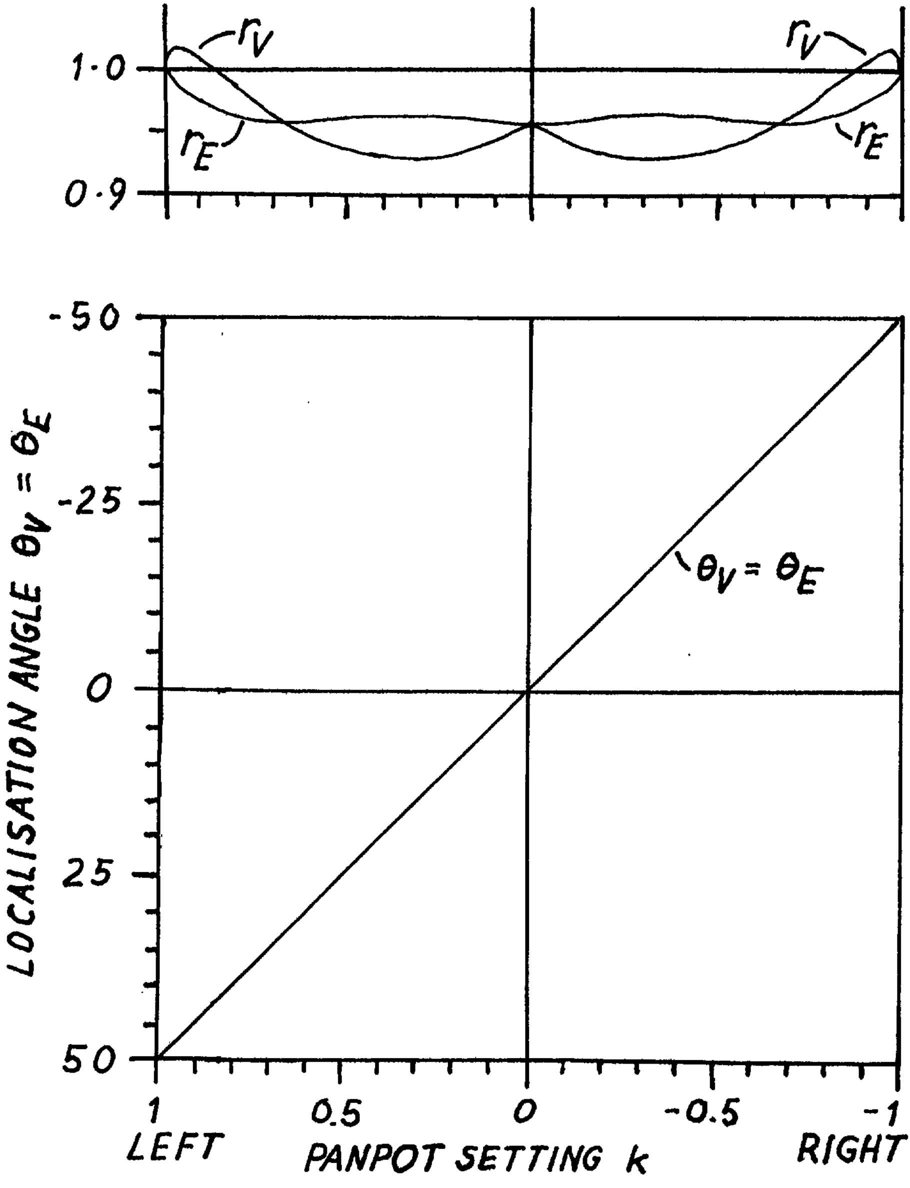

Figure 17. Localisation parameters r_V, Θ_V, r_E, Θ_E of 4-speaker panpot law of table 5 and fig. 16 based on piecewise 3-channel optimal panpot law for 4-speaker layout with $\Theta_4 = 3\Theta_5 = 50^o$.