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Wave Field Synthesis, Adaptive Wave Field Synthesis and Ambisonics using decentralized transformed control: potential applications to *sound field reproduction* and *active noise control*

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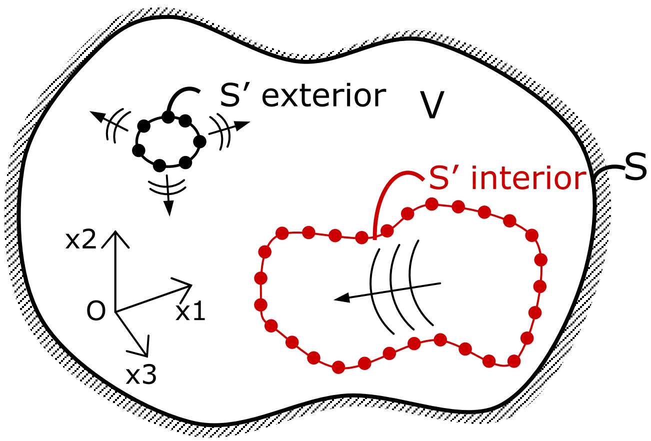
Introduction

Sound field reproduction

- Sound field reproduction finds applications in sound reproduction, experimental acoustics & active control.



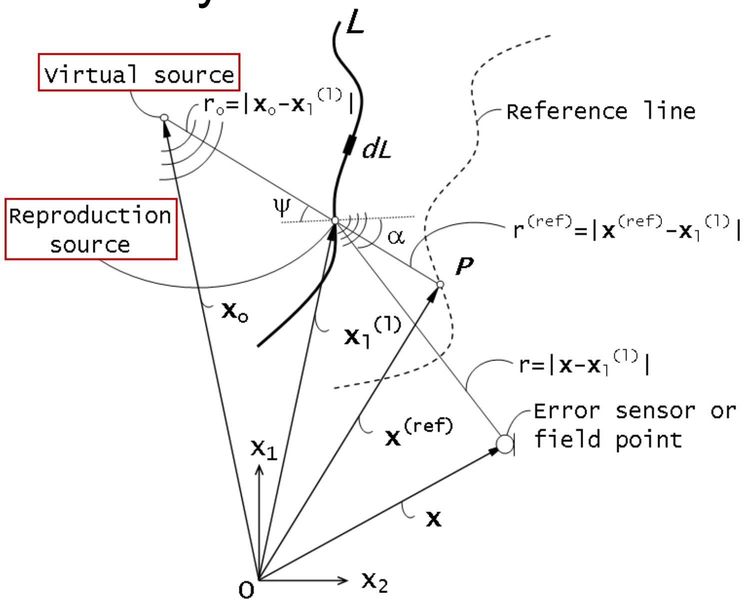
- We focus on the **interior** problem: sound field reproduction **inside** a given region surrounded by acoustical sources.

- Research objectives (presentation content)
  1) Background: WFS and Ambisonics review.
  2) Definition of Adaptive Wave Field Synthesis (AWFS).
  3) Evaluation of the AWFS behavior using Singular Value Decomposition (SVD).
  4) Description of the possible application of AWFS using SVD along with adaptive signal processing.

Review: Wave Field Synthesis (WFS)

Huygens and Kirchhoff-Helmholtz

- ❒ Wave Field Synthesis has been developed in the late 80s by Berkhout.



- ❒ It is based on the integral formulation of the Huygens principle, that is the Kirchhoff-Helmholtz integral.

- ❒ Some WFS simplifications …
  - ⇨ **<u>free-field situation for the reproduction space</u>;**
  - ⇨ 2D line source in the horizontal plane;
  - ⇨ stationary-phase approximation and
  - ⇨ **<u>open-loop architecture</u>**;

lead to the general WFS operators $Q_{WFS}(x,\omega)$ in the frequency domain:

$$Q_{WFS}(\mathbf{x}_l^{(l)},\omega) =$$

$$-A(\omega)j\sqrt{\frac{jk}{2\pi}}\cos\Psi\frac{e^{jkr_o}}{\sqrt{r_o}}\sqrt{r^{(ref)}/(r^{(ref)}+r_o)}\Delta_l$$

Review: Ambisonics

Fourier-Bessel LMS wave field reconstruction

- Ambisonics is from Gerzon in the 70s.
- Assuming a 2D target field defined as a plane wave

$$p^{(im)}(R,\phi,\omega) = A(\omega)e^{jkR\cos(\phi-\theta)}$$

- with a Fourier-Bessel expansion

$$A(\omega)J_0(kR) + 2A(\omega)\sum_{n=1}^{\infty} j^n J_n(kR)\cos(n(\phi-\theta))$$

- the lower order coefficients match simple low order microphone directivity (monopole, dipoles, etc.)

$$p^{(im)}(R,\phi) = A\mathbf{c}^T\mathbf{h}$$

$$\mathbf{c}^T = \sqrt{2}\begin{bmatrix} 1/\sqrt{2} & \cos(\theta) & \sin(\theta) & \dots & \cos(n\theta) & \sin(n\theta) & \dots \end{bmatrix}$$

- Assuming a 2D reproduced field for each reproduction source with the corresponding Fourier-Bessel expansion

$$p^{(rep)}(R,\phi) = \sum_{l=1}^{L} A_l \mathbf{c}_l^T \mathbf{h} = \mathbf{A}^T\mathbf{C}^T\mathbf{h}$$

- Ambisonics seek for an **LMS solution** for the reproduction source amplitudes through pseudo-inversion

$$\mathbf{A} = A\mathbf{C}^{\dagger}\mathbf{c}$$

- these are the standard and general Ambisonics equations which are used with an **open-loop architecture** using a direct free-field simplified model …

Adaptive Wave Field Synthesis

(AWFS: A new concept for sound field rep.)

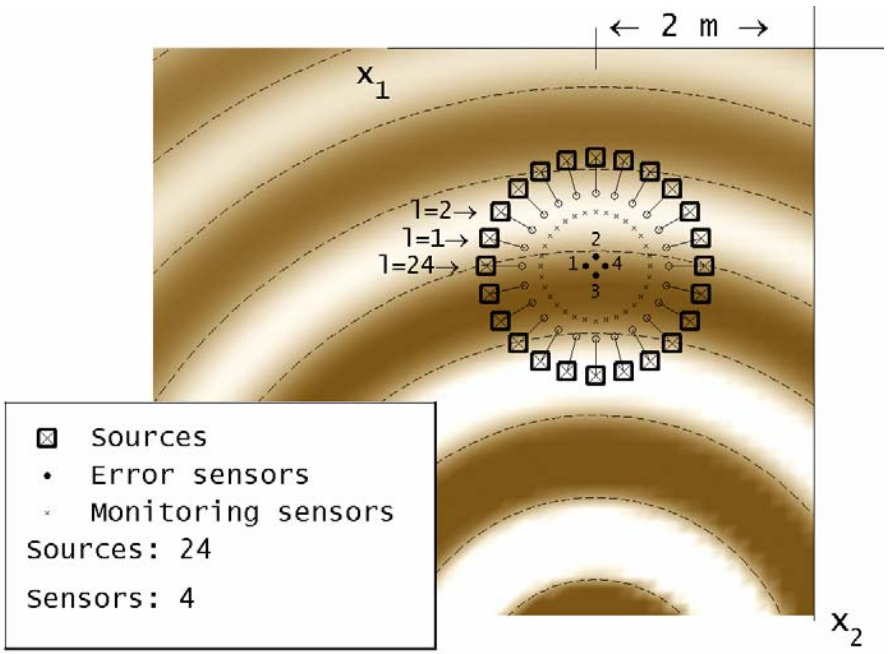
- Adaptive Wave Field Synthesis is physically designed through a cost function which includes **performance index and WFS departure penalty**:

$$J_{AWFS} = \mathbf{e}^H\mathbf{e} + \lambda(\mathbf{q} - \mathbf{q}_{WFS})^H(\mathbf{q} - \mathbf{q}_{WFS})$$

- The optimal solution is found using the optimal source strengths

$$\mathbf{q}^{(opt)} = \left[\mathbf{Z}^H\mathbf{Z} + \lambda\mathbf{I}\right]^{-1}\left[\mathbf{Z}^H\mathbf{p}^{(im)} + \lambda\mathbf{q}_{WFS}\right]$$

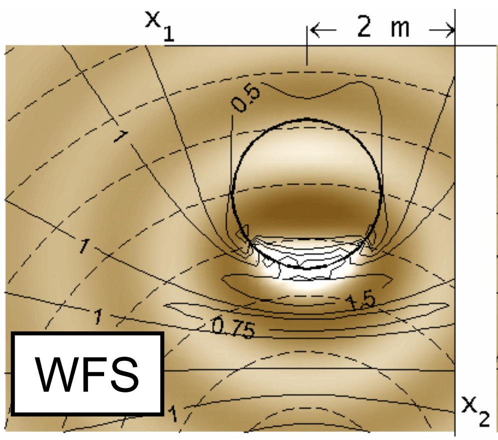
- For the simulations, the system includes a circular source array and a compact sensor array.
- The target field $p^{(im)}$ at 220Hz:

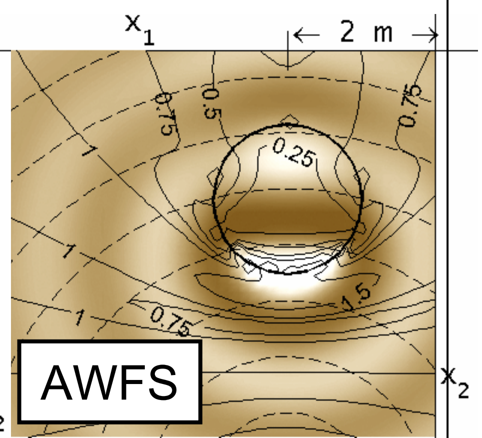


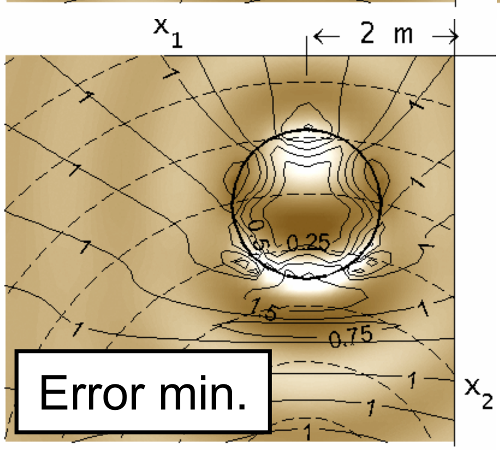
Simulation results

From WFS to AWFS to Ambisonics

- ❒ Reproduced fields at 220Hz …

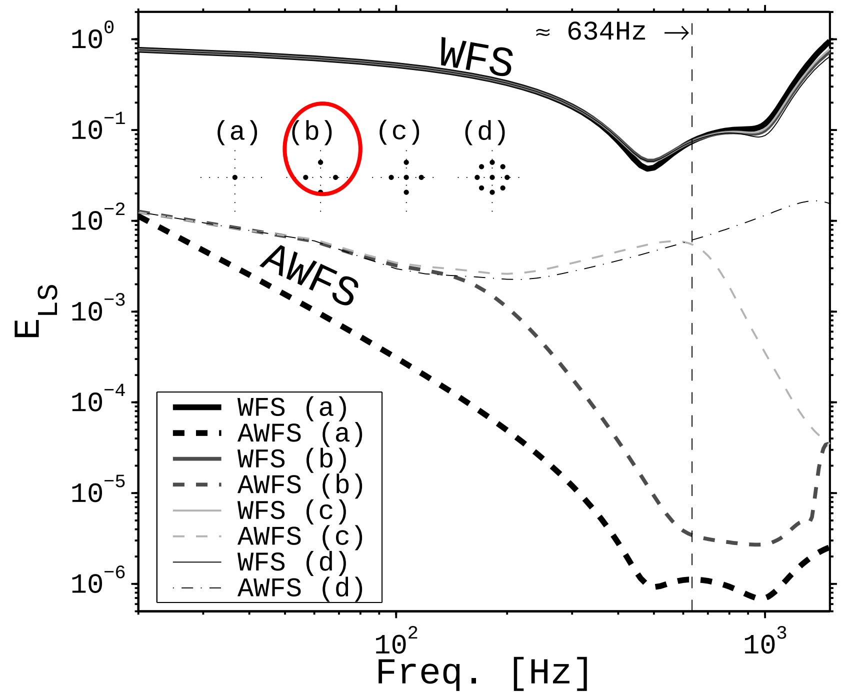






- ❒ WFS: Wave Field Synthesis
- ❒ AWFS: Adaptive WFS with λ=50.
- ❒ Regularized error minimization.

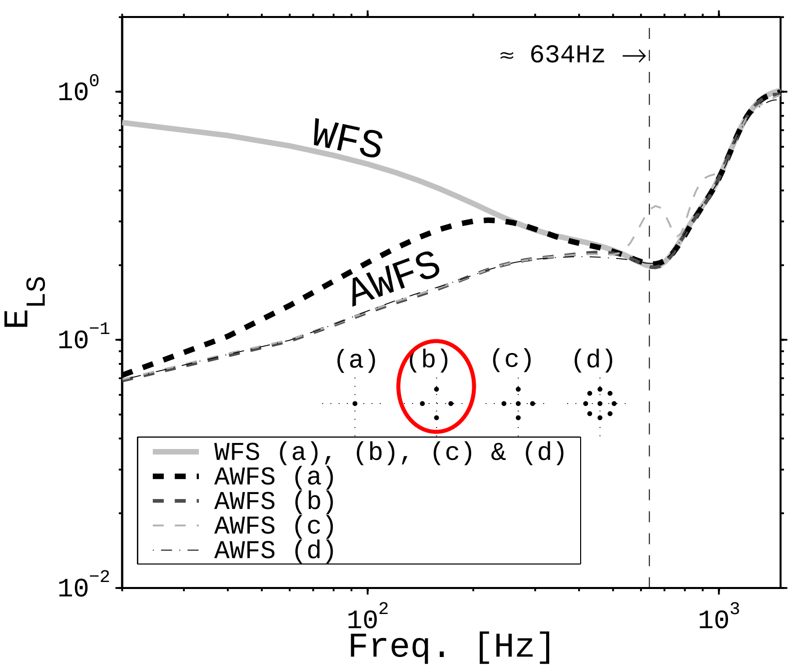
- ❒ Mean reproduction error with WFS and AWFS as function of frequency …



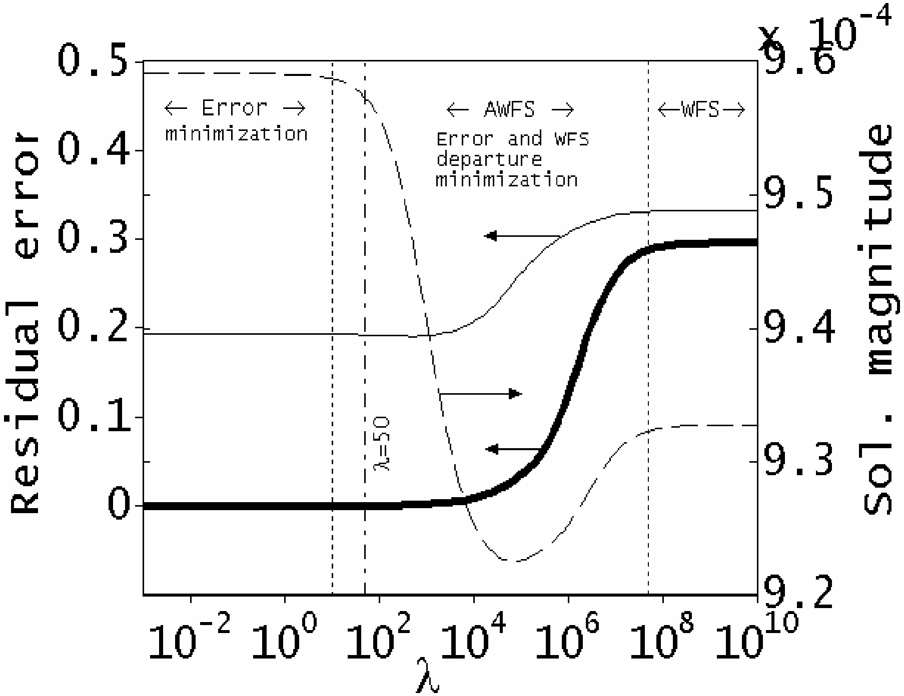
Simulation results

From WFS to AWFS to Ambisonics

- Mean reproduction **monitoring** error reduction with WFS and AWFS as function of frequency …



- According to the penalization parameter λ, one can move from WFS to AWFS and to closed-loop modified Ambisonics …



Current works

Adaptive DSP for AWFS

- FXLMS adaptive scheme for MIMO AWFS (1. Error, 2. cost function, 3. adaptation, 4. convergence)

1

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{R}(n)\mathbf{w}(n)$$

2

$$J_{AWFS} = E[\mathbf{e}^T(n)\mathbf{e}(n)] + \gamma \mathbf{w}^T(n)\mathbf{w}(n) + \\ + \lambda(\mathbf{w}(n) - \mathbf{w}_{WFS})^T(\mathbf{w}(n) - \mathbf{w}_{WFS})$$

3

$$\mathbf{w}(n+1) = (1 - \alpha(\gamma + \lambda))\mathbf{w}(n) + \\ + \alpha\lambda\mathbf{w}_{WFS} + \alpha\hat{\mathbf{R}}^T(n)\mathbf{e}(n)$$

4

$$\boxed{\mathbf{w}_\infty = \frac{E[\hat{\mathbf{R}}^T(n)\mathbf{d}(n)] + \lambda\mathbf{w}_{WFS}}{E[\hat{\mathbf{R}}^T(n)\mathbf{R}(n)] + (\gamma + \lambda)\mathbf{I}}}$$

- FELMS adaptive scheme for MIMO AWFS (1. Error, 2. cost function, 3. adaptation)

1

$$\mathbf{e}(n) = \mathbf{d}(n) - \sum_{j=0}^{J-1}\sum_{i=0}^{I-1}\mathbf{G}_j\mathbf{w}_i x(n-i-j)$$

2

$$J_{AWFS} = \mathrm{tr}[E[\mathbf{e}(n)\mathbf{e}^T(n)]] + \\ + \lambda\left[\sum_{i=0}^{I-1}\mathrm{tr}[[\mathbf{w}_i(n) - \mathbf{w}_{WFS_i}][\mathbf{w}_i(n) - \mathbf{w}_{WFS_i}]^T]\right]$$

3

$$\mathbf{w}_i(n+1) = (1 - \alpha\lambda)\mathbf{w}_i(n) + \\ + \alpha\mathbf{f}(n-J)x(n-i-J) + \alpha\lambda\mathbf{w}_{WFS_i}$$

Z Singular value decomposition

Consequences for AWFS

- The plant SVD (Singular Value Decomposition) gives

$$\mathbf{Z} = \mathbf{U}_{M\times M}\mathbf{\Sigma}_{M\times L}\mathbf{V}^H_{L\times L}$$

$$\tilde{\mathbf{p}}^{(rep)} = \mathbf{\Sigma}\tilde{\mathbf{q}}$$

$$\tilde{\mathbf{p}}^{(im)} = \mathbf{U}^H\mathbf{p}^{(im)},\ \tilde{\mathbf{p}}^{(rep)} = \mathbf{U}^H\mathbf{p}^{(rep)},\ \tilde{\mathbf{q}} = \mathbf{V}^H\mathbf{q}$$

- The solution $q^{(opt)}$ can be expressed as a linear combination of orthogonal source modes: **<u>independent control of the source modes</u>** ...

$$\mathbf{q}^{(opt)} = \sum_{i=1}^{L} \tilde{q}_i \mathbf{v}_i$$

- With SVD, the source modes contribution q_i are expressed as follow

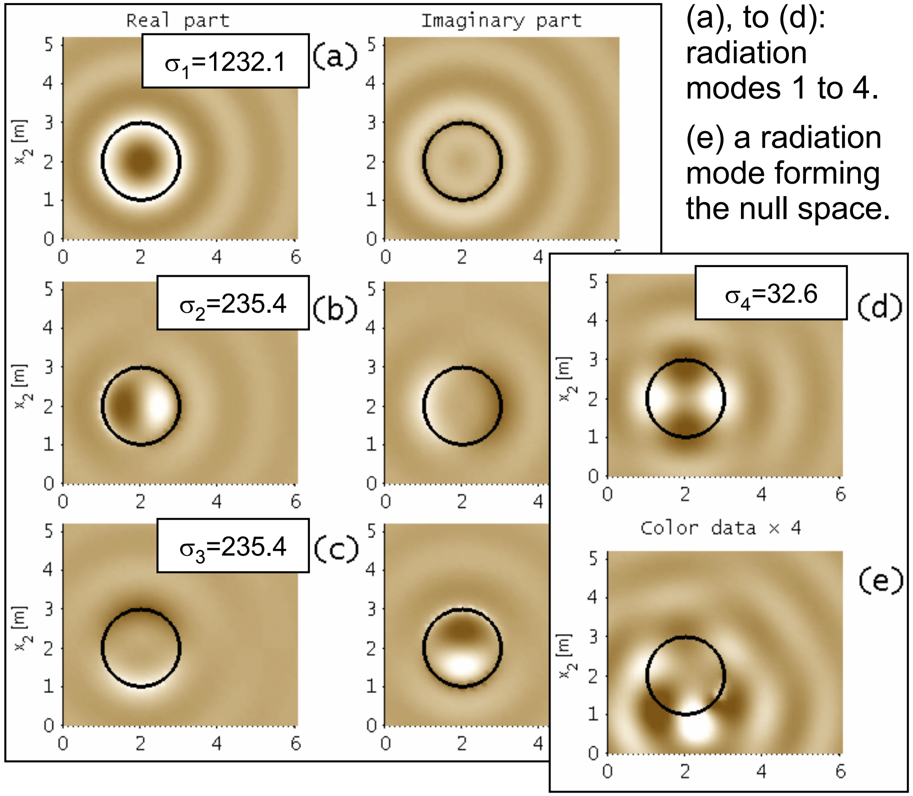
$$\tilde{q}_i = \begin{cases} \frac{\sigma_i}{(\sigma_i^2+\lambda_i)}\tilde{p}_i^{(im)} + \frac{\lambda_i}{(\sigma_i^2+\lambda_i)}\mathbf{v}_i^H\mathbf{q}_{WFS}, & \forall\, i \le r \le M \\ \mathbf{v}_i^H\mathbf{q}_{WFS}, & \forall\, \lambda_i > 0\,, \forall\, i > r \text{ and } i \le L \\ 0, & \forall\, \lambda_i = 0, \forall\, i > r \text{ and } i \le L \end{cases}$$

- This shows that any MIMO AWFS system have redundancy.
- Simplified AWFS with few microphones: more practical and includes the WFS solution by projection on the **Z** null space.

Z Singular value decomposition

Consequences for AWFS

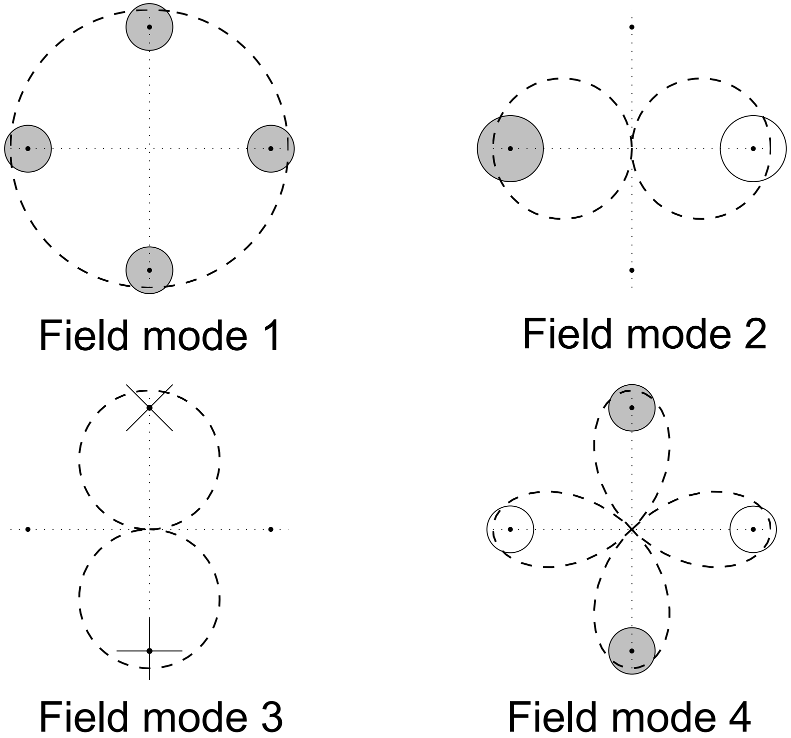
- The fields (220Hz) produced by the source modes: Pressure, pressure gradients, tesseral quadrupole >>> **Ambisonics**.



(a), to (d): radiation modes 1 to 4.

(e) a radiation mode forming the null space.

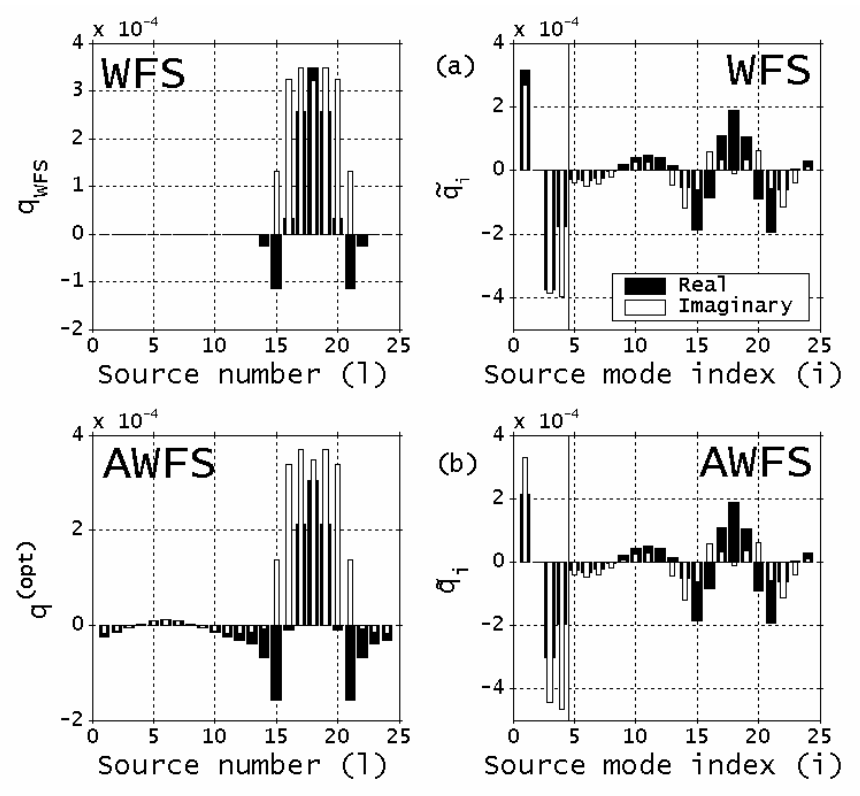
- The corresponding field modes (220Hz) effectively give simple directive patterns (like for Ambisonics) …



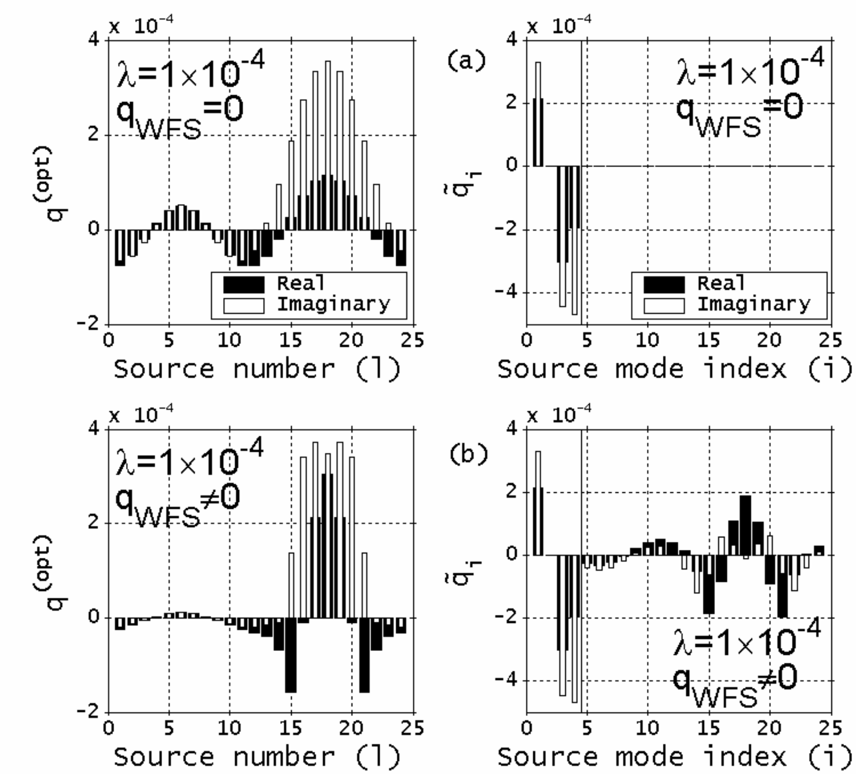
Z Singular value decomposition

Consequences for AWFS

- The WFS solution in comparison with AWFS solution (in space ← and transformed →)

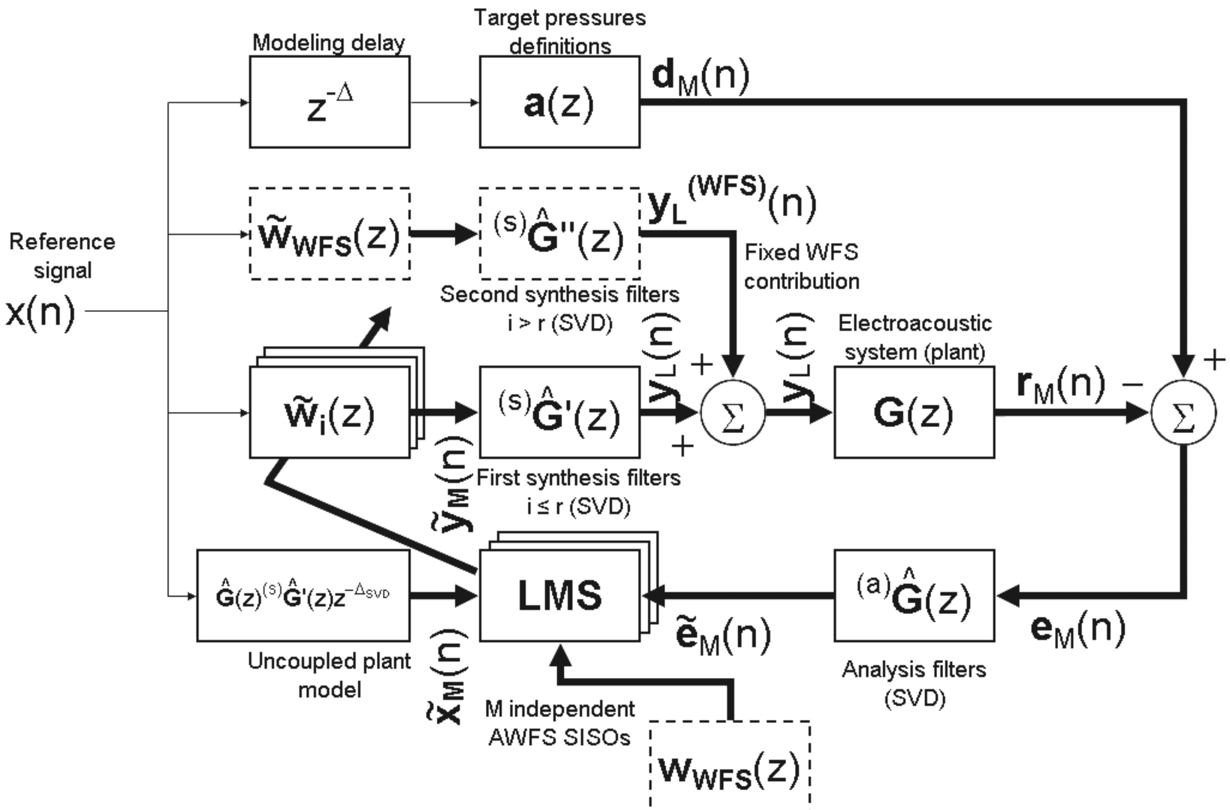


- Illustrative proof of the WFS solution projection on the null space with λ different of 0 …



Current works

Adaptive DSP for AWFS



- Independent radiation mode control and adaptive DSP (current work) …
- Related to "Principal-Component LMS" …
- SVD time-domain filters have been implemented for **broadband SVD decoupling** …

Discussion, conclusion and future works

- AWFS summary:
  1. A cost function to minimize the reproduction error and the departure from the WFS solution.
  2. Independent solution for each radiation mode (pressure, pressure gradients, etc.) on the basis of the plant SVD.
  3. Possible independent adaptive DSP implementation.
  4. **AWFS solution = optimal control solution (closed loop Ambisonics) + WFS solution projected on the transfer matrix null space.**

- Future and current works are devoted to **adaptive algorithms for independent radiation mode control** according to the plant matrix SVD.
- This is now being implemented in DSP simulations before creating a prototype for experiments with such algorithms …
- Other topics of interests related to AWFS: active noise control and inverse problems in acoustics using *a priori* solutions (like the WFS sol.).

Abstract

Wave Field Synthesis, Adaptive Wave Field Synthesis And Ambisonics Using Decentralized Transformed Control: Potential Applications To Sound Field Reproduction And Active Noise Control

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References: http://www3.sympatico.ca/philippe_aubert_gauthier/biblio_1.html

Sound field reproduction finds applications in music or audio reproduction and experimental acoustics. The objective is to reproduce a sound field in a real reproduction environment. Wave Field Synthesis (WFS) is a known open-loop technology which assumes that the reproduction environment is anechoic. Classical WFS thus does not perform well in a real reproduction space. Previous work has suggested that it is physically possible to reproduce a progressive wave field in-room situation using active control approaches. In this presentation, a formulation of Adaptive Wave Field Synthesis (AWFS) introduces practical possibilities for an adaptive sound field reproduction combining WFS and active control (with WFS departure penalization) with a limited number of error sensors. AWFS includes WFS and closed-loop “Ambisonics” as limiting cases. This leads to the modification of multichannel Filtered-Reference Least-Mean-Square (FXLMS) and Filtered-Error LMS (FELMS) adaptive algorithms for AWFS. Possible decentralization for adaptive control of sound field reproduction is introduced on the basis of sources and sensors radiation modes. Such decoupling may lead to decentralized control of source strength distributions and may reduce computational burden of the FXLMS and FELMS for AWFS. [Work funded by NSERC, NATEQ, Université de Sherbrooke and VRQ.]