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Ambisonic Loudspeaker Arrays

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ABSTRACT

The Ambisonic system is one of very few surround sound systems which offers the promise of reproducing full three-dimensional (periphonic) audio. It can be shown that arrays configured as regular polyhedra can allow the recreation of an accurate sound field at the center of the array. But the regular polyhedric shape can be impractical for real everyday usage because the requirement that the listener have his head located at the center of the array forces the location of the lower speakers to be beneath the floor, or even the location of a loudspeaker directly beneath the listener. This is obviously impracticable, especially in domestic applications. Likewise, it is typically the case that the width of the array is larger than can be accommodated within the room boundaries.

The infeasibility of such arrays is a primary reason why they have not been more widely deployed. The intent of this work is to explore the efficacy of alternative array shapes for both horizontal and periphonic reproduction.

1. THE PROBLEM

A problem with implementation of three-dimensional surround sound is that it is difficult to put a 3D loudspeaker array into typical rooms. Typical domestic rooms are longer than they are wide and have lower ceilings than the next smallest dimension. Thus, radially symmetric loudspeaker arrays do not easily fit into the oblong rectangular rooms. The arrays tend to be larger than can be easily fit into a typical domestic space. One possible criterion for a good reproduction array (BS.1116) is that all of the loudspeakers should be located more than 2 meters from the listener, and that they should be no closer than 1 meter to a wall. This might lead to a requirement for a room that is 6 meters wide and has a 6 meter ceiling height, a requirement that is unlikely to be met in a typical domestic situation.

a reproduction array which he called the 'birectangle', an array in which there is a horizontal rectangular array of loudspeakers and a second vertical rectangular array of loudspeakers. An implementation of this has the horizontal array configured as left front, right front, left rear, right rear, and the vertical array configured as front up, front down, back up, and back down. This comes much closer to the practical goal of actually being able to fit the loudspeaker array into the listening room. Another approach is to use two rings of loudspeakers, one located above the horizontal plane and one located an equivalent distance below the horizontal plane. This leads to such arrays as the double square, the double hexagon, double octagon, and so on.

Because typical domestic rooms are narrower and less tall than would be desired for a 3D array, an obvious first step is to squash the arrays in the width and height dimension such that the overall shape more closely corresponds with the room. As long as the speakers remain in the same diametrically-opposed relationship to each other, an Ambisonic decoder can easily be devised for each possible configuration. Decoders can also be derived for more asymmetrical loudspeaker arrays, although with some greater mathematical difficulty and a possible increase in instability.

One approach to the fit problem is to use "warped" diametrically opposed arrays in combination with correction of the time delay and amplitude to restore the correct relationships between the loudspeaker signals as they combine acoustically at the listening position.

In this example the effective width (not accounting for the finite size of the loudspeakers) has been reduced from approximately 3.46 meters for the regular hexagon to only 2.2 meters for the warped hexagon. In this case, because the radius has remained constant for both arrangements, Gerzon's diametrical decoder theorem applies and an optimal decoder can easily be calculated. Because the loudspeakers subtend a significantly smaller horizontal angular space, the amount of Y (left-right) signal that will be required will be proportionately larger and X (front-back) will be proportionately smaller. This could lead to a problem with system overload if the width of the array is made to be too narrow.

A more serious problem is that, while the low-frequency regime decoding for the warped array can be made to be identical to that obtained with the regular hexagonal array, the high-frequency regime decoding will not be as good. It will not be possible to achieve a high value of the energy vector r_E for sources to the sides of the listener. However there is a continuum of performance between the regular hexagon and the highly compressed version shown here. That is, the loudspeaker array need only be warped by the amount necessary to allow it to fit in the room. The function of the layout control in dedicated four-speaker Ambisonic decoders accomplishes a similar function.

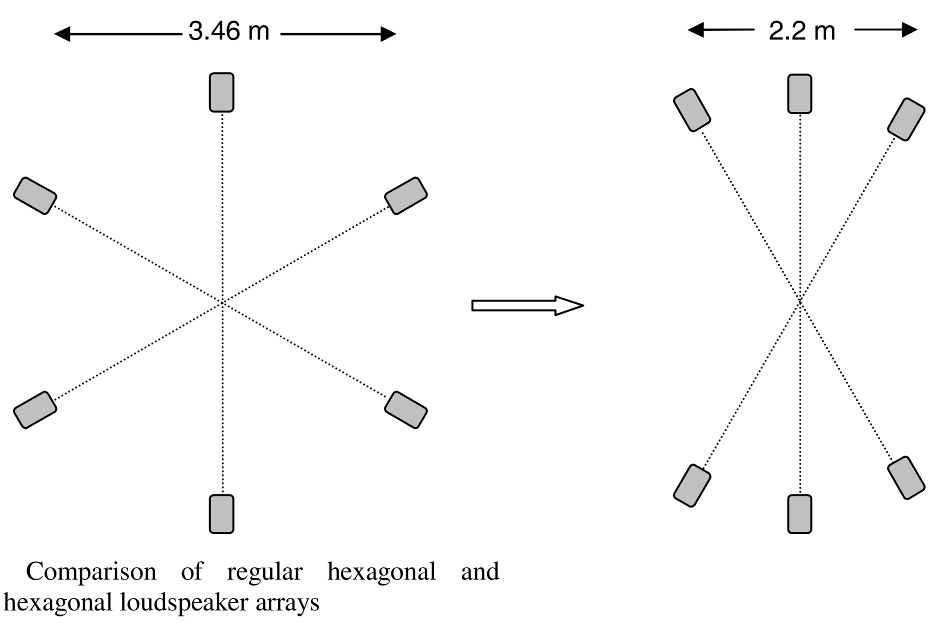


Fig. nn: Comparison of regular hexagonal and "warped" hexagonal loudspeaker arrays

It should be noted that the warped hexagon array is highly useful because it maps onto the variants of the ITU BS.775 array which is nominally used for presently successful surround sound reproduction systems. Although the BS.775 recommendation has the surround loudspeakers located at ±110 degrees, this specification is not commonly met in domestic installations, precisely for the reason that it does not easily fit into ordinary domestic rooms.

The hexagonal array can also be warped by bringing in the side loudspeakers, but without changing their angular disposition relative to the listener:

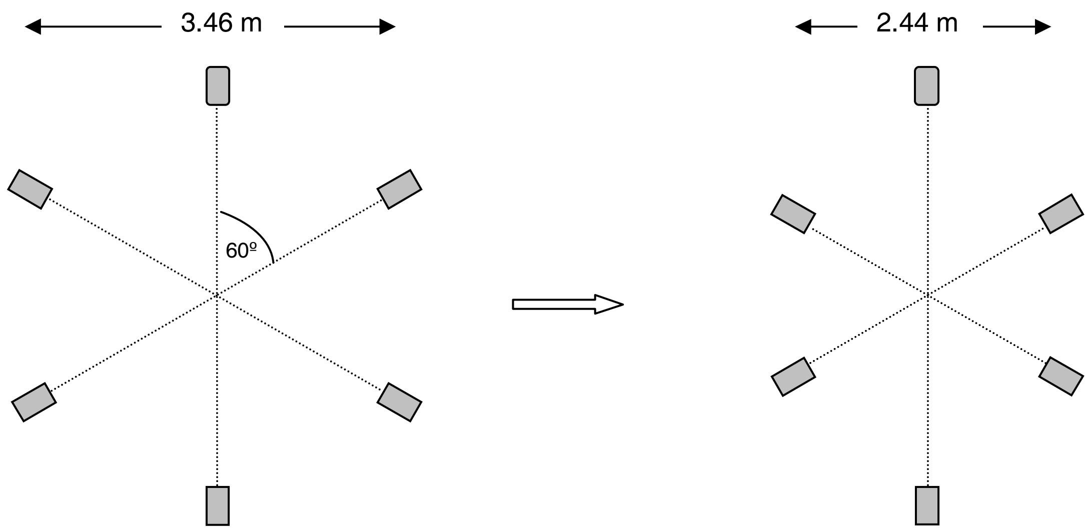


Figure nn: Warping the array and maintaining the angles

In this system, the side loudspeakers have simply been moved in by a factor of √2/2 while maintaining the same angular displacement as the regular hexagonal array. The width of the warped array is 2.44 meters as opposed to 3.46 meters for the regular hexagon array. Since the side loudspeakers are closer, the sound from them arrives sooner and at a higher level than in the regular hexagonal array. In this particular example the radius of the side loudspeakers has been decreased from 2 meters to 1.414 meters, which gives a 3 dB increase in level and a (2-1.414)/343m/s = 1.708 ms earlier arrival for sounds from the side loudspeakers. This can easily be corrected in the decoder.

Because the angular relationships have been retained from the regular hexagonal array, and because the time and intensity have been corrected in the decoder, the direct field from the loudspeakers will be exactly the same at the center as in the un-warped array, except for the proximity effect.

The proximity effect has to do with the relationship between the pressure and the particle velocity in the acoustic waves as the observation point moves closer to the source. At large distances from the source the pressure and particle velocity component of the wave are in phase, but nearer the source there is a reactive component to the acoustic radiation that causes the phase of the particle velocity to approach 90 degrees. At low frequencies the listener can be considered to be in the near field of the loudspeakers, and the phase shift between the pressure and velocity components is different for the closer loudspeaker positions relative to the farther loudspeaker positions. For correct reproduction of the velocity components, the low frequencies should be compensated with a filter that has the reverse characteristic of the proximity effect.

1.1. Regular Polyhedra

Perhaps the most obvious candidates for Periphonic Ambisonic arrays are the regular polyhedra; cubic, dodecahedral, and so on.

1.2. Cylindrical arrays

There is a family of practical periphonic Ambisonic reproduction arrays in which a horizontal loudspeaker array is mirrored through the horizontal plane to create an array of loudspeakers on a vertical cylinder. The following figure depicts one such loudspeaker array:

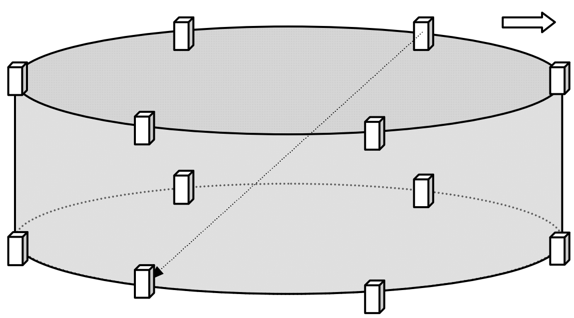
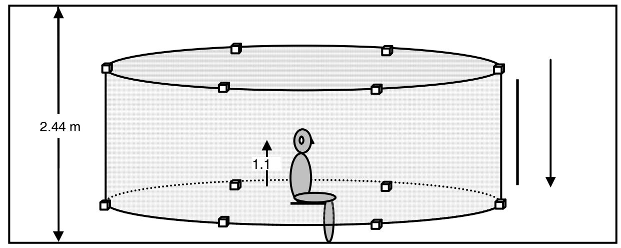
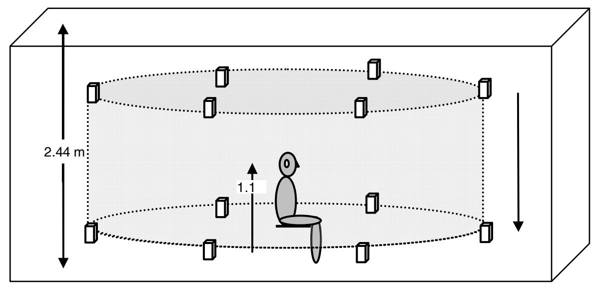


Figure nn: Cylindrical loudspeaker array.

In this array each loudspeaker is diametrically opposed to another, but the array is not any sort of regular polyhedron. The sides of the array are relatively unpopulated, so r_E will not be maximized in all directions. Also, for most practical installations the height of the room will be a limitation, so the Z contribution from the loudspeakers will be low, requiring that the decoder emphasize Z, with an attendant increased potential for system overload. Fortunately, for recordings made in normal environments the Z-channel has the lowest levels of the four B-format signals, so requiring the loudspeakers to output greater amounts of Z doesn't have a great penalty associated with it.

It can be seen in the following figure just how well such arrays can be adjusted to accommodate the generally rectangular dimensions of ordinary rooms:





The ears of an average-height listener are located approximately 1 to 1.1 meters above the floor. Since this is considerably less than one-half way between the ceiling and floor, it is either necessary to locate the two rings of loudspeakers asymmetrically with respect to the room boundaries (for instance, to place the lower loudspeakers on the floor), or to warp the signals fed to the loudspeakers in such a way as to compensate for the fact that the listener is not in the center of the array. The distance between the two rings is an additional constraint, in that the less distance there is, the greater the proportion of 'Z' will be needed to recreate the proper velocity field at the listening position. Small heights for the loudspeaker array may lead to excessive signal levels for the individual loudspeakers when programs containing significant proportions of height information are reproduced.

1.3. Energy Vector analysis of various loudspeaker arrays:

Gerzon described a group of theorems [3] which predict the localization performance of the system of loudspeakers and decoder. The basic premise is to calculate what he described as the velocity vector and energy vector [refer to BLaH paper at this convention], and the localization is assumed to be optimum (perfect?) if the directions of the vectors coincide (no confusing localization cues), and the magnitudes are both unity. Unfortunately, as he showed, although the magnitude of the velocity vector can be made to be unity and the direction of the velocity vector can be made to coincide with the energy vector, but the magnitude of the energy vector will only be unity if the sound comes exactly from the direction of a loudspeaker and if only that loudspeaker is being driven. In general, the localization cannot be made to be perfect, and the best that can be done is to maximize the energy vector. Given that this is a compromise situation, it may be desirable to improve the localization in one direction at the expense of another. Furthermore, it will be seen that the attempt to expand the reproduction from the horizontal plane to the whole sphere results in a general worsening of the energy vector and a decrease in the quality of the localization.

Since the goal of this paper is to describe practical ways to implement periphony, it will be seen that the selection of a loudspeaker array, beyond just the practical restrictions described above, must address the necessary compromises, in both the loudspeaker array and the decoder, which determine the eventual direction-dependant localization performance.

2. REAL ROOM EXPERIMENTS

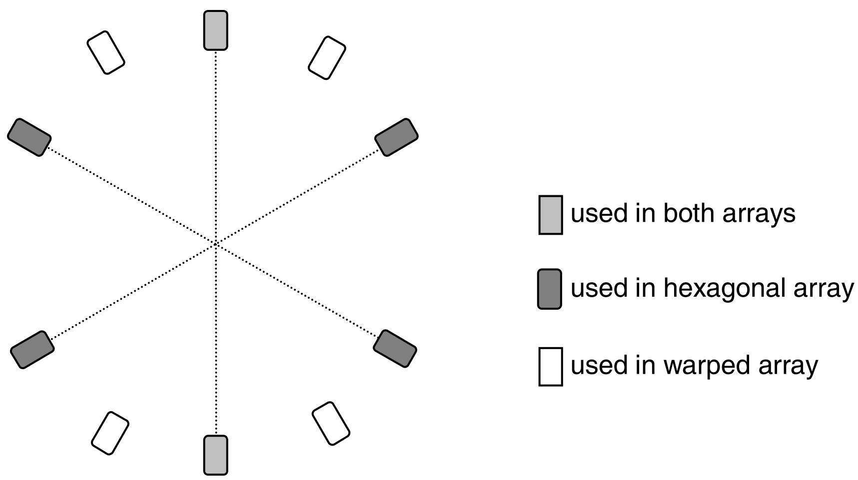
Several of the ideas discussed above were tested in the listening room CMAP, located in a private residence. This listening room was designed to correspond more closely to the acoustics of normal domestic rooms than most listening rooms in research facilities, while at the same time to not have certain limitations, such as excessive environmental noise or excessive low-frequency reverberation. CMAP has the following measured characteristics:

- Dimensions: 6.7m X 4.5m X 2.44 m
- Construction: drywall on wood framing
- Noise floor: less than NC10
- Reverberation time: 0.30 seconds

Audio playback is from a silent PC fitted with a 16-channel audio interface, and twelve JBL LSR25p loudspeakers are supported on movable vertical poles which allow them to be located at any height between the ceiling and floor.

Experimental Protocol:

Given a sufficiently large listening room, it is possible to set up a traditional regular polygonal or polyhedral speaker array, and then to progressively move the loudspeakers away from their regular positions into increasingly warped positions. It is also possible in some cases to superimpose the two loudspeaker arrays. That is, both arrays can be constructed at the same time and in the same place.



Regular hexagonal and warped hexagonal loudspeaker array with 2-meter radius

Several methods for switching between the two playback systems can be envisioned. One of them would be to have a real-time Ambisonic decoder which allowed the possibility of switching between the two decoder modes and simultaneously switching the outputs to the appropriate loudspeakers where required. A second method is to do a non-real-time decode of all of the program material, using decoders appropriate to the two loudspeaker layouts. The decoded signals can then be mapped to the appropriate loudspeakers within the ten loudspeaker array by placing them within the appropriate channels of ten-channel interleaved PCM files. Switching between the two decoder arrays can then be achieved by switching the playback between one of two files, the one containing the decoded signals for the regular hexagon, or the one containing the decode for the warped hexagon.

To ensure that the test conditions are as nearly equal as possible, the following conditions must be met:

- The loudspeakers must be as near to identical as possible.
- The loudspeaker positions must be exactly those for which the decoder has been calculated

One of the principle virtues of Ambisonic reproduction is that it is, in principle, isotropic. The symmetries of the recording (or encoding) system and of the reproduction array ensure that the quality of localization is the same in every direction.

Test signals were derived which allow the evaluation of the localization characteristics of the reproduction. These test signals were encoded rather than recorded in order to limit the evaluation to the characteristics of the reproduction system and not of the microphone system.

- perceived localization in the direction of the encoded direction
- quality of reproduction; does the timbre or the size of the sound change with position?

These qualities were first evaluated monochromatically. Gaussian bursts were synthesized at frequencies on standard one-third octave intervals from 100 Hz to 2000 Hz, and then panned in small (how small?) increments around the circle according to the first-order Ambisonic encoding formulas:

$W = 0.707s$

$X = \cos\theta s$

$Y = \sin\theta s$

Accuracy of localization was evaluated by comparing the perceived location of the burst to the panned location. It is known (Blauert) that perceived location is not the same as the real location for real sources at the side. Rather than compare the perceived localization from phantom images to that of the desired localization, it is more reasonable to compare the phantom image localization with that which would have been experienced from a real source.

Could compare the sound of a panned burst with that of the same burst reproduced through a loudspeaker located at the panned position. This would presumably result in the sound of the panned burst being heard as more diffuse than the one emitted only from a single loudspeaker.

3. PERIPHONIC ARRAYS

The term ‘Periphonic’ is used to describe surround sound systems which also reproduce height. It might be said that such systems are the only ones that should be called ‘Surround Sound’, and in this paper we will refer to systems without height as being ‘horizontal-only surround sound’ systems.

Periphonic systems have the room-related difficulties described above, and also are generally subject to the difficulty that they cannot extend above the ceiling, or below the floor. Some few systems at research institutes have steel-mesh floors with loudspeakers mounted below them.

In principle, any loudspeaker array with loudspeakers located both above and below the listener could be used for periphony, but Gerzon’s ‘diagonal decoder theorem’ shows that the principle localization parameters are guaranteed to coincide, with an appropriate decoder, if each loudspeaker has a diagonal opposite.

3.1. Cylindrical Arrays

Any horizontal loudspeaker array for which there is an ambisonic decoder can be converted into a periphonic array by creating a double array with identical arrays equally offset above and below the horizontal plane. For the case where there are an odd number of loudspeakers one of the arrays will have to be rotated by 180 degrees in order to create an opposing loudspeaker.

A decoder for the cylindrical array can be created from the decoder for the original horizontal array by dividing the coefficients by two, to compensate for the doubling of the number of loudspeakers, and by multiplying the horizontal coefficients by $1/\cos\theta$ to compensate for the decreasing projection onto the horizontal plane, and finally by creating vertical components proportional to $1/N\sin\theta$.

Rectangular Arrays

A variety of interesting periphonic loudspeaker arrays can be created by placing rectangles of loudspeakers into different planes which intersect the listener's position. Gerzon described one such array which he called the bi-rectangle. As he described it, it was comprised of two rectangles, one in the horizontal plane and one in a vertical plane going from left to right. It appears like this, as he drew it:

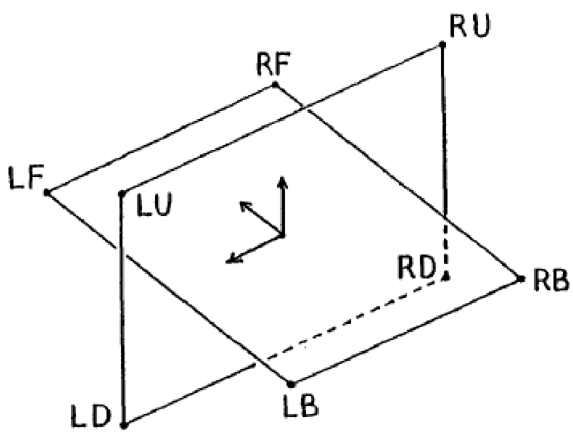


Figure nn: the bi-rectangle (after reference [2])

This idea can be extended by the addition of another rectangle of loudspeakers to create a 'tri-rectangle', as depicted in the following figure:

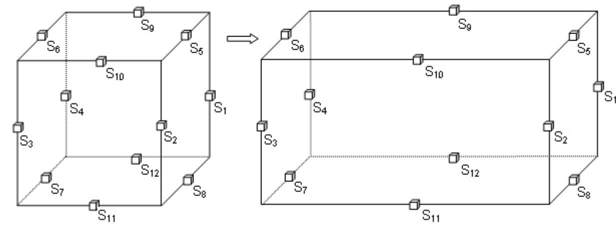
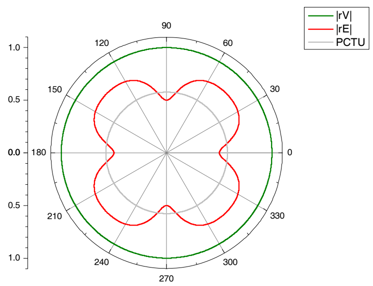


Figure nn: Tri-rectangle loudspeaker array

The addition of the additional group of four loudspeakers helps with some problems of the bi-rectangle array, particularly that the magnitude of r_E can be rather low in the directions where there are no loudspeakers, and the fact that the bi-rectangle is mostly inappropriate for reproducing programs of second-order content.

Energy optimization for second order periphonic decoders

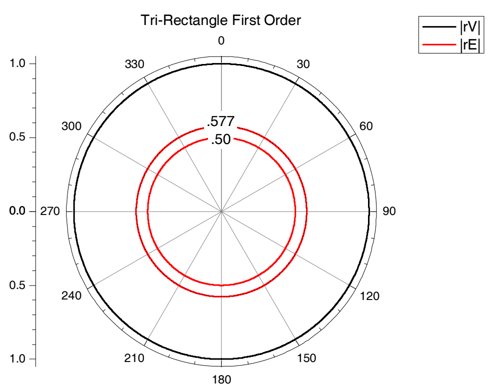


alter the ratio of W to (X,Y) and (U,V) while observing the value of $|r_E|$.

This would need to be normalized to give equal loudness at low and high frequencies.

For tri-rectangle second-order decoders, starting with the decoder for a cubic tri-rectangle array:

Plotting the magnitude of r_E around the horizontal circle for the first order case:



These results are identical to those for other first-order periphonic regular arrays. If the analysis is extended to second order decoders, the following result is obtained:

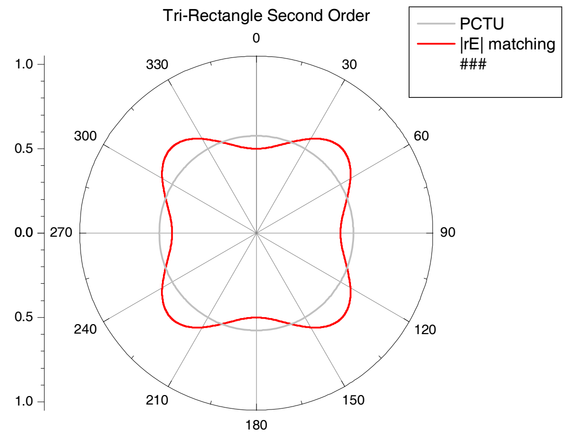


Figure nn: Angular variation of $|\vec{r}_E|$ for tri-rectangle

It can be seen that the magnitude of $\vec{r}_E$ varies with direction, being highest in the direction of the loudspeakers in the horizontal plane.

A result similar to what is described using 'shelf filters' for horizontal-only decoders can be obtained by altering the ratios between the zeroth, first, and second-order components [W (X,Y,Z) R (U,V)], while observing the value of $|\vec{r}_E|$.

For the case of the tri-rectangle, using second order decoding results in the magnitude of $\vec{r}_E$ changing significantly with direction; specifically for a cubic tri-rectangle, and using matching decoding, the magnitude of $\vec{r}_E$ is .5 in the cardinal directions, and 0.714 in the horizontal direction of the speakers. Altering the ratios of zeroth to the first to the second order coefficients (W/(X,Y)/(U,V)) can be used to maximize $|\vec{r}_E|$. However, the optimum ratios to maximize $\vec{r}_E$ are different in different directions, so a decision must be made as to which directions are most important, or to maximize $\vec{r}_E$ globally as opposed to in one direction. As it happens, a choice can be made which maximizes r_E in the cardinal directions, where it needs it most, and nearly maximizes it in the direction of the speakers:

The optimum values of coefficients for maximizing $\vec{r}_E$ in the cardinal directions are approximately 1.00, 0.76, 0.50. This gives the following result:

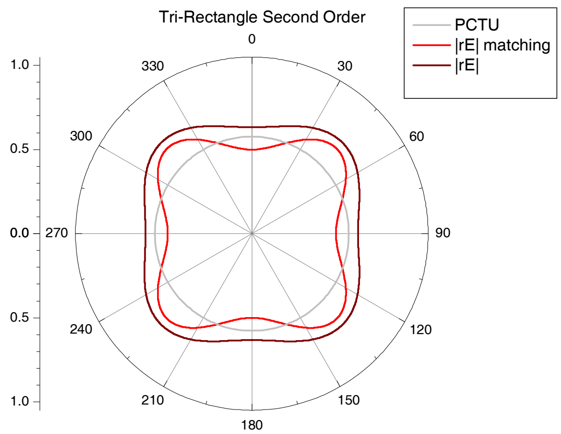


Figure nn: Angular variation of $|\vec{r}_E|$ for tri-rectangle with coefficients optimized to maximize $\vec{r}_E$

In which $\vec{r}_E$ is increased in every direction, but the greatest increase is in the cardinal directions, where it was worst before optimization.

4. DISCUSSION

5. CONCLUSIONS

6. REFERENCES

[1] Gerzon, M.; "Practical Periphony", Preprint No. 1571, 65th AES Convention, (February 1980)

[2] Gerzon, M.; "Surround Sound Psychoacoustics", Wireless World, Vol. No.

[3] Gerzon, M.; "General Metatheory of Auditory Localization", Preprint No. 3306, 92nd AES Convention, (March 1992)

[4] Gerzon, M. "Periphony; With-Height Sound Reproduction", JAES, Vol. 21, No. 1, pp. 2-10, (December 1972)

[5] Theiβ, B.; Hawksford. M. "Localization Experiments in Three-Dimensional Sound Reproduction", Preprint 4156, 100th AES Convention (1996 May)

[6] Craven, P., Posting to Sursound List, https://mail.music.vt.edu/mailman/listinfo/sursound Tue, 27 Apr 2004 14:14:30 +0100

[7] Blauert, Jens; "Spatial Hearing: The Psychophysics of Human Sound Localization", ISBN 0262024136 Published MIT Press (1997)

[8] http://www.muse.demon.co.uk/ref/speakers.html

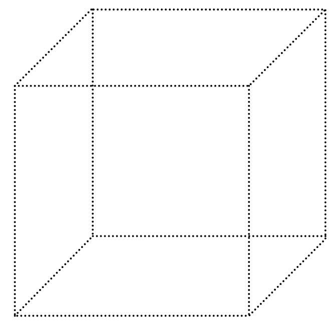
[9] http://iem.at/~zmoelnig/publications/cube_en/

[10]

7. APPENDIX 1: PERIPHONIC DECODER COEFFICIENTS

The following tables give the decoder coefficients used for the periphonic loudspeaker arrays described in this paper. The general method of presentation follows conventions used by Furse [furse]. Array coordinates are Cartesian coordinates on the unit circle, starting with the loudspeaker directly ahead of the listener, and progressing clockwise from that point.

Cube



Array coordinates

	X	Y	Z
1	0.5774	0.5774	-0.5774
2	0.5774	-0.5774	-0.5774
3	-0.5774	-0.5774	-0.5774
4	-0.5774	0.5774	-0.5774
5	0.5774	0.5774	0.5774
6	0.5774	-0.5774	0.5774
7	-0.5774	-0.5774	0.5774
8	-0.5774	0.5774	0.5774

First Order decoder coefficients

	W	X	Y	Z
1	0.1768	+0.2165	+0.2165	-0.2165
2	0.1768	+0.2165	-0.2165	-0.2165
3	0.1768	-0.2165	-0.2165	-0.2165
4	0.1768	-0.2165	+0.2165	-0.2165
5	0.1768	+0.2165	+0.2165	+0.2165
6	0.1768	+0.2165	-0.2165	+0.2165
7	0.1768	-0.2165	-0.2165	+0.2165
8	0.1768	-0.2165	+0.2165	+0.2165

Second Order decoder coefficients

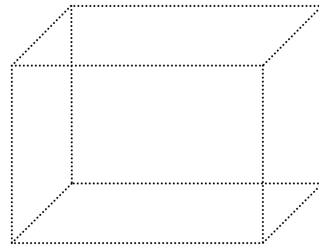
	W	X	Y	Z	R	S	T	U	V
1	0.1768	+0.2165	+0.2165	-0.2165	0.0000	-0.1875	-0.1875	0.0000	+0.1875
2	0.1768	+0.2165	-0.2165	-0.2165	0.0000	-0.1875	+0.1875	0.0000	-0.1875
3	0.1768	-0.2165	-0.2165	-0.2165	0.0000	+0.1875	+0.1875	0.0000	+0.1875
4	0.1768	-0.2165	+0.2165	-0.2165	0.0000	+0.1875	-0.1875	0.0000	-0.1875
5	0.1768	+0.2165	+0.2165	+0.2165	0.0000	+0.1875	+0.1875	0.0000	+0.1875
6	0.1768	+0.2165	-0.2165	+0.2165	0.0000	+0.1875	-0.1875	0.0000	-0.1875
7	0.1768	-0.2165	-0.2165	+0.2165	0.0000	-0.1875	-0.1875	0.0000	+0.1875
8	0.1768	-0.2165	+0.2165	+0.2165	0.0000	-0.1875	+0.1875	0.0000	-0.1875

W 0 dB (1.0000)
X, Y -1.25 dB (0.8660)
U, V -6.02 dB (0.5000)

Cuboid, 30 degree

Array coordinates

	X	Y	Z
1	0.6124	0.6124	-0.5000
2	0.6124	-0.6124	-0.5000
3	-0.6124	-0.6124	-0.5000
4	-0.6124	0.6124	-0.5000
5	0.6124	0.6124	0.5000
6	0.6124	-0.6124	0.5000
7	-0.6124	-0.6124	0.5000
8	-0.6124	0.6124	0.5000



First Order decoder coefficients

	W	X	Y	Z
1	0.1768	0.2041	0.2041	-0.2500
2	0.1768	0.2041	-0.2041	-0.2500
3	0.1768	-0.2041	-0.2041	-0.2500
4	0.1768	-0.2041	0.2041	-0.2500
5	0.1768	0.2041	0.2041	0.2500
6	0.1768	0.2041	-0.2041	0.2500
7	0.1768	-0.2041	-0.2041	0.2500
8	0.1768	-0.2041	0.2041	0.2500

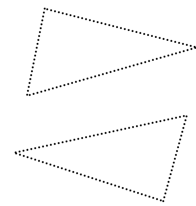
Second Order decoder coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.1768	0.2041	0.2041	-0.2500	0.0000	-0.2041	-0.2041	0.0000	0.1667
2	0.1768	0.2041	-0.2041	-0.2500	0.0000	-0.2041	0.2041	0.0000	-0.1667
3	0.1768	-0.2041	-0.2041	-0.2500	0.0000	0.2041	0.2041	0.0000	0.1667
4	0.1768	-0.2041	0.2041	-0.2500	0.0000	0.2041	-0.2041	0.0000	-0.1667
5	0.1768	0.2041	0.2041	0.2500	0.0000	0.2041	0.2041	0.0000	0.1667
6	0.1768	0.2041	-0.2041	0.2500	0.0000	0.2041	-0.2041	0.0000	-0.1667
7	0.1768	-0.2041	-0.2041	0.2500	0.0000	-0.2041	-0.2041	0.0000	0.1667
8	0.1768	-0.2041	0.2041	0.2500	0.0000	-0.2041	0.2041	0.0000	-0.1667

Bi-triangle

Array coordinates

	X	Y	Z
1	0.7500	0.4330	0.5000
2	0.0000	0.8660	-0.5000
3	-0.7500	0.4330	0.5000
4	-0.7500	-0.4330	-0.5000
5	0.0000	-0.8660	0.5000
6	0.7500	-0.4330	-0.5000



First Order Decoder Coefficients

	W	X	Y	Z
1	0.2357	0.3333	0.1925	0.3333
2	0.2357	0.0000	0.3849	-0.3333
3	0.2357	-0.3333	0.1925	0.3333
4	0.2357	-0.3333	-0.1925	-0.3333
5	0.2357	0.0000	-0.3849	0.3333
6	0.2357	0.3333	-0.1925	-0.3333

Second Order Decoder Coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.2357	0.3333	0.1925	0.3333	0.0000	0.0000	0.0000	0.2222	0.3849
2	0.2357	0.0000	0.3849	-0.3333	0.0000	0.0000	0.0000	-0.4444	0.0000
3	0.2357	-0.3333	0.1925	0.3333	0.0000	0.0000	0.0000	0.2222	-0.3849
4	0.2357	-0.3333	-0.1925	-0.3333	0.0000	0.0000	0.0000	0.2222	0.3849
5	0.2357	0.0000	-0.3849	0.3333	0.0000	0.0000	0.0000	-0.4444	0.0000
6	0.2357	0.3333	-0.1925	-0.3333	0.0000	0.0000	0.0000	0.2222	-0.3849

Bi-rectangle

Array Coordinates

	X	Y	Z
LF	0.7071	0.7071	0.0000
LB	0.7071	-0.7071	0.0000
RB	-0.7071	0.7071	0.0000
RF	-0.7071	-0.7071	0.0000
CFU	0.7071	0.0000	0.7071
CFD	0.7071	0	-0.7071
CBD	-0.7071	0	0.7071
CBU	-0.7071	0	-0.7071

First Order Decoder Coefficients

	W	X	Y	Z
LF	0.1768	+0.1768	+0.3536	0.0000
LB	0.1768	-0.1768	+0.3536	0.0000
RB	0.1768	-0.1768	-0.3536	0.0000
RF	0.1768	+0.1768	-0.3536	0.0000
CFU	0.1768	+0.1768	0.0000	+0.3536
CFD	0.1768	+0.1768	0.0000	-0.3536
CBD	0.1768	-0.1768	0.0000	-0.3536
CBU	0.1768	-0.1768	0.0000	+0.3536

Second Order Decoder Coefficients

	W	X	Y	Z	R	S	T	U	V
LF	0.1178	+0.1768	+0.3536	-0.0000	-0.3333	-0.0000	+0.0000	+0.0000	+0.2500
RF	0.1178	+0.1768	-0.3536	-0.0000	-0.3333	-0.0000	+0.0000	+0.0000	-0.2500
LB	0.1178	-0.1768	+0.3536	+0.0000	-0.3333	+0.0000	+0.0000	+0.0000	-0.2500
RB	0.1178	-0.1768	-0.3535	+0.0000	-0.3333	+0.0000	+0.0000	+0.0000	+0.2500
CFU	0.2357	+0.1768	+0.0000	+0.3536	+0.3333	+0.2500	+0.0000	+0.0000	+0.0000
CFD	0.2357	+0.1768	+0.0000	-0.3536	+0.3333	-0.2500	+0.0000	+0.0000	+0.0000
CBD	0.2357	-0.1768	+0.0000	-0.3536	+0.3333	+0.2500	+0.0000	+0.0000	+0.0000
CBU	0.2357	-0.1768	+0.0000	0.3536	+0.3333	-0.2500	+0.0000	+0.0000	+0.0000

Bi-rectangle 30

Array Coordinates

	X	Y	Z
LF	0.7071	0.7071	0.0000
LB	0.7071	-0.7071	0.0000
RB	-0.7071	0.7071	0.0000
RF	-0.7071	-0.7071	0.0000
CFU	0.8660	0.0000	0.5000
CFD	0.8660	0.0000	-0.5000
CBD	-0.8660	0.0000	0.5000
CBU	-0.8660	0.0000	-0.5000

First Order Decoder Coefficients

	W	X	Y	Z
LF	0.1768	0.1414	0.3536	0.0000
LB	0.1768	0.1414	-0.3536	0.0000
RB	0.1768	-0.1414	0.3536	0.0000
RF	0.1768	-0.1414	-0.3536	0.0000
CFU	0.1768	0.1732	-0.0000	0.5000
CFD	0.1768	0.1732	-0.0000	-0.5000
CBD	0.1768	-0.1732	0.0000	0.5000
CBU	0.1768	-0.1732	0.0000	-0.5000

Second Order Decoder Coefficients

	W	X	Y	Z	R	S	T	U	V
LF	0.3536	0.1414	0.3536	0	0.0000	-0.0000	0.0000	-0.3333	0.2500
RF	0.3536	0.1414	-0.3536	0	0.0000	-0.0000	0.0000	-0.3333	-0.2500
LB	0.3536	-0.1414	0.3536	0	0.0000	0.0000	0.0000	-0.3333	-0.2500
RB	0.3536	-0.1414	-0.3536	0	0.0000	0.0000	0.0000	-0.3333	0.2500
CFU	0	0.1732	0.0000	0.5000	0.0000	0.2887	0.0000	0.3333	0.0000
CFD	0	0.1732	0.0000	-0.5000	0.0000	-0.2887	0.0000	0.3333	0.0000
CBD	0	-0.1732	0.0000	0.5000	0.0000	-0.2887	0.0000	0.3333	0.0000
CBU	0	-0.1732	0.0000	-0.5000	0.0000	0.2887	0.0000	0.3333	0.0000

Tri-Rectangle Array (45 degree or cuboid array)

Array Coordinates

	X	Y	Z
1	0.7071	0.0000	0.7071
2	0.7071	0.0000	-0.7071
3	-0.7071	0.0000	0.7071
4	-0.7071	0.0000	-0.7071
5	0.7071	0.7071	0.0000
6	0.7071	-0.7071	0.0000
7	-0.7071	0.7071	0.0000
8	-0.7071	-0.7071	0.0000
9	0.0000	0.7071	0.7071
10	0.0000	0.7071	-0.7071
11	0.0000	-0.7071	0.7071
12	0.0000	-0.7071	-0.7071

First Order Decoder Coefficients

	W	X	Y	Z
1	0.1179	0.1768	0.0000	0.1768
2	0.1179	0.1768	0.0000	-0.1768
3	0.1179	-0.1768	0.0000	0.1768
4	0.1179	-0.1768	0.0000	-0.1768
5	0.1179	0.1768	0.1768	0.0000
6	0.1179	0.1768	-0.1768	0.0000
7	0.1179	-0.1768	0.1768	0.0000
8	0.1179	-0.1768	-0.1768	0.0000
9	0.1179	0.0000	0.1768	0.1768
10	0.1179	0.0000	0.1768	-0.1768
11	0.1179	0.0000	-0.1768	0.1768
12	0.1179	0.0000	-0.1768	-0.1768

Second Order Decoder Coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.1179	0.1768	0.0000	0.1768	0.1667	0.2500	0.0000	0.2500	+0.0000
2	0.1179	0.1768	0.0000	0.1768	0.1667	0.2500	0.0000	0.2500	+0.0000
3	0.1179	-0.1768	0.0000	0.1768	0.1667	0.2500	0.0000	0.2500	+0.0000
4	0.1179	-0.1768	0.0000	-0.1768	0.1667	0.2500	0.0000	0.2500	+0.0000
5	0.1179	0.1768	0.1768	0.0000	-0.3333	0.0000	0.0000	0.0000	+0.2500
6	0.1179	0.1768	-0.1768	0.0000	-0.3333	0.0000	0.0000	0.0000	-0.2500
7	0.1179	-0.1768	0.1768	0.0000	-0.3333	0.0000	0.0000	0.0000	-0.2500
8	0.1179	-0.1768	-0.1768	0.0000	-0.3333	0.0000	0.0000	0.0000	+0.2500
9	0.1179	0.0000	0.1768	0.1768	0.1667	0.0000	0.2500	-0.2500	+0.0000
10	0.1179	0.0000	0.1768	-0.1768	0.1667	0.0000	-0.2500	-0.2500	+0.0000
11	0.1179	0.0000	-0.1768	0.1768	0.1667	0.0000	-0.2500	-0.2500	+0.0000
12	0.1179	0.0000	-0.1768	-0.1768	0.1667	0.0000	0.2500	-0.2500	+0.0000

Tri-Rectangle 30 degree Array:

Array coordinates

	X	Y	Z
1	0.8660	0.0000	0.5000
2	0.8660	0.0000	-0.5000
3	-0.8660	0.0000	0.5000
4	-0.8660	0.0000	-0.5000
5	0.7071	0.7071	0.0000
6	0.7071	-0.7071	0.0000
7	-0.7071	0.7071	0.0000
8	-0.7071	-0.7071	0.0000
9	0.0000	0.8660	0.5000
10	0.0000	0.8660	-0.5000
11	0.0000	-0.8660	0.5000
12	0.0000	-0.8660	-0.5000

First order decoder coefficients

	W	X	Y	Z
1	0.1178	0.1732	0.0000	0.2500
2	0.1178	0.1732	0.0000	-0.2500
3	0.1178	-0.1732	0.0000	0.2500
4	0.1178	-0.1732	0.0000	-0.2500
5	0.1178	0.1414	0.1414	0.0000
6	0.1178	0.1414	-0.1414	0.0000
7	0.1178	-0.1414	0.1414	0.0000
8	0.1178	-0.1414	-0.1414	0.0000
9	0.1178	0.0000	0.1732	0.2500
10	0.1178	0.0000	0.1732	-0.2500
11	0.1178	0.0000	-0.1732	0.2500
12	0.1178	0.0000	-0.1732	-0.2500

Second order decoder coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.2357	0.1732	0.0000	0.2500	0.3333	0.2887	0.0000	0.1667	0.0000
2	0.2357	0.1732	0.0000	-0.2500	0.3333	-0.2887	0.0000	0.1667	0.0000
3	0.2357	-0.1732	0.0000	0.2500	0.3333	-0.2887	0.0000	0.1667	0.0000
4	0.2357	-0.1732	0.0000	-0.2500	0.3333	0.2887	0.0000	0.1667	0.0000
5	-0.1178	0.1414	0.1414	0.0000	-0.6666	0.0000	0.0000	0.0000	0.2500
6	-0.1178	0.1414	-0.1414	0.0000	-0.6666	0.0000	0.0000	0.0000	-0.2500
7	-0.1178	-0.1414	0.1414	0.0000	-0.6666	0.0000	0.0000	0.0000	-0.2500
8	-0.1178	-0.1414	-0.1414	0.0000	-0.6666	0.0000	0.0000	0.0000	0.2500
9	0.2357	0.0000	0.1732	0.2500	0.3333	0.0000	0.2887	-0.1667	0.0000
10	0.2357	0.0000	0.1732	-0.2500	0.3333	0.0000	-0.2887	-0.1667	0.0000
11	0.2357	0.0000	-0.1732	0.2500	0.3333	0.0000	-0.2887	-0.1667	0.0000
12	0.2357	0.0000	-0.1732	-0.2500	0.3333	0.0000	0.2887	-0.1667	0.0000

Double hexagon (45 degree)

Array coordinates

	X	Y	Z
1	0.6123	0.3536	0.7071
2	0.0000	0.7071	0.7071
3	-0.6123	0.3536	0.7071
4	-0.6123	-0.3536	0.7071
5	0.0000	-0.7071	0.7071
6	0.6123	-0.3536	0.7071
7	0.6123	0.3536	-0.7071
8	0.0000	0.7071	-0.7071
9	-0.6123	0.3536	-0.7071
10	-0.6123	-0.3536	-0.7071
11	0.0000	-0.7071	-0.7071
12	0.6123	-0.3536	-0.7071

First order decoder coefficients

	W	X	Y	Z
1	0.1179	+0.2041	+0.1179	+0.1179
2	0.1179	+0.0000	+0.2357	+0.1179
3	0.1179	-0.2041	+0.1179	+0.1179
4	0.1179	-0.2041	-0.1179	+0.1179
5	0.1179	+0.0000	-0.2357	+0.1179
6	0.1179	+0.2041	-0.1179	+0.1179
7	0.1179	+0.2041	+0.1179	-0.1179
8	0.1179	+0.0000	+0.2357	-0.1179
9	0.1179	-0.2041	+0.1179	-0.1179
10	0.1179	-0.2041	-0.1179	-0.1179
11	0.1179	+0.0000	-0.2357	-0.1179
12	0.1179	+0.2041	-0.1179	-0.1179

Second order decoder coefficients

	W	X	Y	Z	R	S	T	U	V
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

Double hexagon (30 degree)

Array coordinates

	X	Y	Z
1	0.7500	0.4330	0.5000
2	0.0000	0.8660	0.5000
3	-0.7500	0.4330	0.5000
4	-0.7500	-0.4330	0.5000
5	0.0000	-0.8660	0.5000
6	0.7500	-0.4330	0.5000
7	0.7500	0.4330	-0.5000
8	0.0000	0.8660	-0.5000
9	-0.7500	0.4330	-0.5000
10	-0.7500	-0.4330	-0.5000
11	0.0000	-0.8660	-0.5000
12	0.7500	-0.4330	-0.5000

First order decoder coefficients

	W	X	Y	Z
1	0.1620	0.0625	0.2285	0.2292
2	0.1326	-0.1458	0.1083	0.1875
3	0.1179	-0.1458		
4	0.1326			
5	0.1620			
6	0.1473			
7	0.1326			
8	0.1031			
9	0.0884			
10	0.1031			
11	0.1326			
12	0.1620			

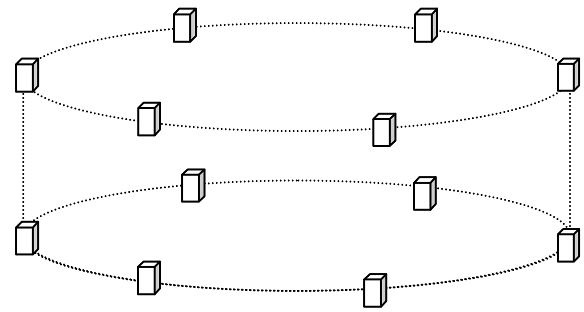
Second order decoder coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.1179								
2	0.1179								
3	0.1179								
4	0.1179								
5	0.1179								
6	0.1179								
7	0.1179								
8	0.1179								
9	0.1179								
10	0.1179								
11	0.1179								
12	0.1179								

Double Octagon (30 degree)

Array coordinates

	X	Y	Z
1	0.8001	0.3314	0.5000
2	0.3314	0.8001	0.5000
3	-0.3314	0.8001	0.5000
4	-0.8001	0.3314	0.5000
5	-0.8001	-0.3314	0.5000
6	-0.3314	-0.8001	0.5000
7	0.3314	-0.8001	0.5000
8	0.8001	-0.3314	0.5000
9	0.8001	0.3314	-0.5000
10	0.3314	0.8001	-0.5000
11	-0.3314	0.8001	-0.5000
12	-0.8001	0.3314	-0.5000
13	-0.8001	-0.3314	-0.5000
14	-0.3314	-0.8001	-0.5000
15	0.3314	-0.8001	-0.5000
16	0.8001	-0.3314	-0.5000



First order decoder coefficients

	W	X	Y	Z
1	0.0884	0.1333	0.0553	0.1250
2	0.0884	0.0552	0.1333	0.1250
3	0.0884	-0.0552	0.1333	0.1250
4	0.0884	-0.1333	0.0552	0.1250
5	0.0884	-0.1333	-0.0552	0.1250
6	0.0884	-0.0552	-0.1333	0.1250
7	0.0884	0.0552	-0.1333	0.1250
8	0.0884	0.1333	-0.0552	0.1250
9	0.0884	0.1333	0.0552	-0.1250
10	0.0884	0.0552	0.1333	-0.1250
11	0.0884	-0.0552	0.1333	-0.1250
12	0.0884	-0.1333	0.0552	-0.1250
13	0.0884	-0.1333	-0.0552	-0.1250
14	0.0884	-0.0552	-0.1333	-0.1250
15	0.0884	0.0552	-0.1333	-0.1250
16	0.0884	0.1333	-0.0552	-0.1250

Second order decoder coefficients

	W	X	Y	Z	R	S	T	U	V
1	0.0884	0.1333	0.0552	0.1250	0.0000	0.1333	0.0552	0.1179	0.1178
2	0.0884	0.0552	0.1333	0.1250	0.0000	0.0552	0.1333	-0.1179	0.1178
3	0.0884	-0.0552	0.1333	0.1250	0.0000	-0.0552	0.1333	-0.1179	-0.1178
4	0.0884	-0.1333	0.0552	0.1250	0.0000	-0.1333	0.0552	0.1179	-0.1178
5	0.0884	-0.1333	-0.0552	0.1250	0.0000	-0.1333	-0.0552	0.1179	0.1178
6	0.0884	-0.0552	-0.1333	0.1250	0.0000	-0.0552	-0.1333	-0.1179	0.1178
7	0.0884	0.0552	-0.1333	0.1250	0.0000	0.0552	-0.1333	-0.1179	-0.1178
8	0.0884	0.1333	-0.0552	0.1250	0.0000	0.1333	-0.0552	0.1179	-0.1178
9	0.0884	0.1333	0.0552	-0.1250	0.0000	-0.1333	-0.0552	0.1179	0.1178
10	0.0884	0.0552	0.1333	-0.1250	0.0000	-0.0552	-0.1333	-0.1179	0.1178
11	0.0884	-0.0552	0.1333	-0.1250	0.0000	0.0552	-0.1333	-0.1179	-0.1178
12	0.0884	-0.1333	0.0552	-0.1250	0.0000	0.1333	-0.0552	0.1179	-0.1178
13	0.0884	-0.1333	-0.0552	-0.1250	0.0000	0.1333	0.0552	0.1179	0.1178
14	0.0884	-0.0552	-0.1333	-0.1250	0.0000	0.0552	0.1333	-0.1179	0.1178
15	0.0884	0.0552	-0.1333	-0.1250	0.0000	-0.0552	0.1333	-0.1179	-0.1178
16	0.0884	0.1333	-0.0552	-0.1250	0.0000	-0.1333	0.0552	0.1179	-0.1178