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Sound field analysis based on generalized prolate spheroidal wave sequences

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ABSTRACT

In this article, an array processing is described to improve the quality of sound field analysis, which aims to extract spatial properties of a sound field. In this domain, the notion of spatial aliasing inevitably occurs due to the finite number of microphones used in the array. It is linked to the Fourier transform of the discrete analysis window, which is constituted of a mainlobe, fixing the resolution achievable by the spatial analysis, and also from sidelobes which degrade the quality of spatial analysis by introducing artifacts not present in the original sound field. A method to design an optimal analysis window with respect to a particular wave vector is presented, aiming to realize the best localization possible in the wave vector domain. Then the efficiency of the approach is demonstrated for several geometrical configurations of the microphone array, on the whole bandwidth of sound fields.

1. INTRODUCTION

Sound field analysis is used in a variety of domains such as the study of vibrating structures, generally to localize the origin of unwanted noise sources, or in sound reproduction systems. It could be divided into two steps: the first one consists to choice a model describing correctly a sound field inside a given analysis area with a set of parameters, and the second one concerns the estimation of the parameters associated to this model. Several possibilities exist to represent a given sound field: in Wave Field Synthesis (WFS), the representation is based on Kirchhoff's integral equation [1], whose parameters are the acoustic pressure and the normal derivative with respect to the surface delimiting the analysis area for all points of the surface; in High Order Ambisonics (HOA), the sound field is represented as an infinite sum of spherical harmonics [2], whose parameters are the coefficients weighting each spherical harmonic. Concerning the estimation of the parameters, there are also two strategies: the first one is uses acoustic models for propagation and acoustic sources, which enable to compute all the parameters of the model in simple cases; the second one concerns the estimation of the parameters in real conditions, assuming that the knowledge of the sound field is limited to a finite number of measures obtained by a microphone array, like in [3], [4], or [5]. It is this last category of problem this article is concerned with. For this category, while the number of measures available is finite, the set of parameters to be estimated grows rapidly with frequency, so that an exact solution exists only for the lower part of the whole frequency bandwidth of sound fields [5][6].

In this paper, an array processing is described to improve the performance of sound field analysis. The processing is based on the multidimensional Fourier transform. An optimal analysis window, weighting the sampled sound field, is designed with a criterion inspired by spectral analysis based on prolate spheroidal wave sequences [7]. The first part of this article recalls some results concerning the propagation of sound, and the multidimensional Fourier transform. In the second part, spatial aliasing, which inevitably occurs when using a microphone array, is interpretated by the Fourier transform of the analysis window, which is composed from a mainlobe and also from sidelobes. A method is proposed to design an analysis window weighting the sampled sound field in order to achieve the best localization possible in the wave vector domain. The efficiency of the approach is demonstrated in the next part using several geometrical array configurations, and it is also shown that good performance is achieved on the whole frequency bandwidth of sound fields. Finally, some concluding remarks and perspectives on this research are made.

2. SOUND FIELD ANALYSIS IN THE CONTINUOUS TIME-SPACE DOMAIN

Sound field analysis lies upon the study of the acoustic pressure field $p(\mathbf{r},t)$, granted that the acoustic pressure is the relevant physical quantity our ear is sensitive to. The purpose of this article is to perform an approximated sound field analysis over an extended area, in order to reconstruct as accurately as possible the initial sound field over the same relative domain, which does not need to be located in the same place than the initial one, by a subsidiary device, such as a loudspeaker array. An efficient analysis tool is the multidimensional Fourier transform, which expands the sound field as a superposition of plane waves. The physical background is introduced in a first part, then some points about the four-dimensional Fourier transform are remembered.

2.1. Physical background

In a domain empty of sources, the sound field is assumed to obey the following second order hyperbolic partial differential equation [8]:

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = 0 \tag{1}$$

The Fourier transform of the above equation, as defined by relation (7), yields:

$$\left(|\mathbf{k}|^2 - \frac{\omega^2}{c^2}\right) P(\mathbf{k},\omega) = 0 \tag{2}$$

Thus, if there are no sources inside the domain of interest, nontrivial solutions are obtained only if the *dispersion relationship* is verified:

$$|\mathbf{k}|^2 = \frac{\omega^2}{c^2} \tag{3}$$

Possible solutions to the wave equation (1) are the plane waves $\exp[\mathrm{i}(\mathbf{k}\cdot\mathbf{r}+\omega t)]$ verifying the dispersion relation (3).

2.2. Mathematical background

Consider the sound field $p(\mathbf{r},t)$ as an element of the set of the tempered distributions of $\mathbb{R}^4$, noted $\mathscr{S}\left(\mathbb{R}^4\right)$ thereafter. This is the most proper set to deal with the four-dimensional Fourier transform that will be introduced later. The set of the tempered distributions $\mathscr{S}\left(\mathbb{R}^4\right)$ is supplied with the following inner product:

$$< f|g >= \iiiint_{(\mathbf{r},t)\in\mathbb{R}^4} f(\mathbf{r},t)\,\overline{g}(\mathbf{r},t)\,\mathrm{d}^3\mathbf{r}\mathrm{d}t \tag{4}$$

where g has to be an element of the dual set of the tempered distributions $\mathscr{S}'\left(\mathbb{R}^4\right)$, that is g is an infinitely smooth function of $\mathbb{R}^4$, which is polynomially bounded with all of its derivatives. The quadruple integral is a mathematical abuse, when dealing with distributions, but is physically convenient for the purpose of this article.

Consider now the family of plane waves:

$$\psi_{\mathbf{k},\omega}(\mathbf{r},t) = \exp[\mathrm{i}(\mathbf{k}\cdot\mathbf{r}+\omega t)]$$

with $(\mathbf{k},\omega) \in \mathbb{R}^4$. This is an orthogonal set from equation (5), which is moreover complete because of the completeness relationship (6):

$$< \psi_{\mathbf{k}_1,\omega_1} | \psi_{\mathbf{k}_2,\omega_2} > \; = \; (2\pi)^4 \, \delta\left(\mathbf{k}_1 - \mathbf{k}_2\right) \delta\left(\omega_1 - \omega_2\right) \quad (5)$$

$$\delta\left(\mathbf{r}-\mathbf{r}_0\right)\delta\left(t-t_0\right) = \frac{1}{(2\pi)^4} \iiiint_{(\mathbf{k},\omega)\in\mathbb{R}^4} e^{\mathrm{i}[\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_0)+\omega(t-t_0)]} \mathrm{d}^3\mathbf{k}\mathrm{d}\omega \quad (6)$$

From the two above equations, we can say that any sound field belonging to $\mathscr{S}'\left(\mathbb{R}^4\right)$ can be expanded onto the plane waves basis. The associated *analysis operator* is nothing else than the four-dimensional Fourier transform[9], or plane wave decomposition:

$$P(\mathbf{k},\omega) = \iiiint_{(\mathbf{r},t)\in\mathbb{R}^4} p(\mathbf{r},t)\, e^{-\mathrm{i}(\mathbf{k}\cdot\mathbf{r}+\omega t)} \mathrm{d}^3\mathbf{r}\mathrm{d}t \qquad (7)$$

The sound field can be perfectly reconstructed in a least-square sense from the knowledge of the Fourier transform $P(\mathbf{k},\omega)$ for all $(\mathbf{k},\omega) \in \mathbb{R}^4$, by using the *synthesis operator*, which is the inverse four-dimensional Fourier transform:

$$p(\mathbf{r},t) = \frac{1}{(2\pi)^4} \iiiint_{(\mathbf{k},\omega)\in\mathbb{R}^4} P(\mathbf{k},\omega)\, e^{+\mathrm{i}(\mathbf{k}\cdot\mathbf{r}+\omega t)} \mathrm{d}^3\mathbf{k}\mathrm{d}\omega \quad (8)$$

Another important result is the Parseval-Plancherel relation, which only applies if f and g are elements of $\mathscr{L}^2\left(\mathbb{R}^4\right)$:

$$\begin{aligned} < f|g > &= \frac{1}{(2\pi)^4} \iiiint_{(\mathbf{k},\omega)\in\mathbb{R}^4} F(\mathbf{k},\omega)\, \overline{G}(\mathbf{k},\omega)\, \mathrm{d}^3\mathbf{k}\mathrm{d}\omega \\ &= < \mathscr{F}(f) || \mathscr{F}(g) > = < F||G > \qquad (9) \end{aligned}$$

where F and G are the Fourier transforms of f and g. This states that the Fourier transform is an isometry.

The sound field model that will be used throughout this article is the knowledge of the Fourier transform of the sound field. At first sight, this requires the knowledge of $P(\mathbf{k},\omega)$ for all $(\mathbf{k},\omega) \in \mathbb{R}^4$. But this initial set of parameters can be firstly reduced to the subset specified by the dispersion relationship (3). And it can be further reduced assuming that the soundfield is bandlimited in the frequency band $|\omega| < \Omega$, and thus also in the wave vector domain inside the sphere of radius $|\mathbf{k}| < \Omega/c = K$.

3. PRACTICAL ESTIMATION OF THE PARAMETERS

The parameters of the sound field model are perfectly known only in theoretical cases, such as sound fields radiated by point sources, with monopole or more complex directivity functions. The topic of this article is the estimation of the parameters associated to the model under practical conditions, where only a finite number of measures is available, provided usually by a microphone array of N elements. Unfortunately, even if the set of parameters has been reduced by taking into account the remarks made in the last paragraph, it is still an uncountable set of parameters of infinite dimension. Granted that only N spatial measures are available, an exact estimation of these parameters is impossible. The finiteness of the microphone array inevitably introduces spatial aliasing, which is the topic of the next paragraph. Some processing is introduced at section 3.2 to perform an approximate estimation of the parameters.

3.1. Spatial aliasing

The only available sound field to be analyzed is the sampled version of the original one. In this article, the problem of spatial sampling is considered, and the time sampling is supposed to be perfectly controlled. Indeed, perfect reconstruction of time-sampled signals is achievable in practical conditions, granted that the sampling frequency is minimum twice of the maximal frequency present in the signal of interest. If the same conditions are imposed to spatial sampling, it would require a density of 2.2 10^6 sensors per m^3 uniformly distributed in the cartesian coordinate system, assuming that the wavenumbers are limited to the domain $|\mathbf{k}| < 2\pi.22050/340$: this is unrealistic in practical considerations.

The spatially sampled sound field is given by the following relation:

$$p_{sam}(\mathbf{r},t) = \sum_{n=1}^{N} \mathscr{V} w_n \delta\left(\mathbf{r}-\mathbf{r_n}\right) p(\mathbf{r},t) = w(\mathbf{r},t)\,.p(\mathbf{r},t) \qquad (10)$$

where $\mathscr{V}$ is a volume of normalization, and w_n is a weight, so that $w(\mathbf{r},t)$ is a finite discrete analysis window, without dimension.

The Fourier transform —which is used as the sound field model— of the sampled sound field is linked to the Fourier transform of the original sound field by the relation:

$$P_{sam}(\mathbf{k},\omega) = W(\mathbf{k},\omega) \star_4 P(\mathbf{k},\omega) \tag{11}$$

where $\star_4$ denotes the symbol of the convolution product along the four dimensions. Indeed, the simple product is transformed into a convolution product by the Fourier transform.

The condition of non aliasing is that the restriction of $W(\mathbf{k},\omega)$ to the domain defined by $\omega < 2\Omega, |\mathbf{k}| < 2K$ is identical to $\delta(\mathbf{k})\,\delta(\omega)$. Otherwise, spatial aliasing occurs. This is typically the case under real conditions, because of the finiteness of the array. Indeed, the Fourier transform of the analysis window is then made up with a main lobe whose spread indicates the resolution of the measure and also with sidelobes.

3.2. Optimal estimation of the parameters

In this article, we want an estimator of the Fourier transform evaluated at a given point $\hat{P}(\mathbf{k_0},\omega_0)$, and we impose it to be independent of the sound field being analyzed $p(\mathbf{r},t)$. Granted that the sound field satisfies the dispersion relationship, it is relevant to apply spatial analysis in frequency subbands, and ideally in monochromatic cases. In this case, the dispersion relationship gives an *a priori* location on the region of the spatial spectrum being excited, which is the sphere of radius $k_0 = |\omega_0|/c$. This enables to drop the frequency dependency of the four-dimensional Fourier transform, and to only consider the three-dimensional spatial Fourier transform, for a given pulsation ω_0. As mentionned before, the weights w_n are the only degrees of adjustment of the analysis system when the geometry of the array has been fixed. The first possibility is to design an analysis window which realizes a tradeoff between resolution and apodization, which has already been investigated in [10]. The corresponding analysis window is designed independently of the Fourier atom $\psi_{\mathbf{k_0}}$ being analyzed. The other possibility is to use different weights w_n according to the wave vector $\mathbf{k_0}$ of interest. Indeed:

$$\begin{aligned}\hat{P}(\mathbf{k_0}) &= < w_{\mathbf{k_0}} p | \psi_{\mathbf{k_0}} > = < p | \overline{w}_{\mathbf{k_0}} \psi_{\mathbf{k_0}} > \\ &= < P || \mathscr{F}\left(\overline{w}_{\mathbf{k_0}} \psi_{\mathbf{k_0}}\right) >\end{aligned} \tag{12}$$

The atom $\psi_{\mathbf{k_0}}$ is not localized in the space domain, but is well localized in the wave vector domain. On the other hand, the element $\overline{w}_{\mathbf{k_0}}\psi_{\mathbf{k_0}}$ is localized in the space domain, so that it is no longer localized in the wave vector domain. So, it seems relevant that the weights $w_n(\mathbf{k}_0)$ have to be chosen in order to achieve the best localization possible in the wave vector domain.

In the following $\mathscr{F}\left(\overline{w}_{\mathbf{k_0}}\psi_{\mathbf{k_0}}\right)$ is noted $W_{\mathbf{k_0}}\Psi_{\mathbf{k_0}}$ but is different from the product of the two Fourier transforms taken separately. The statement of the last paragraph stands as a basis idea to design the weights $w_n(\mathbf{k_0})$, so that it maximizes the following ratio:

$$\lambda = \frac{\iiint_{\mathbf{k}\in \mathrm{S}(\mathbf{k_0},k_{res})} |W_{\mathbf{k_0}}\Psi_{\mathbf{k_0}}(\mathbf{k})|^2 \mathrm{d}^3\mathbf{k}}{\iiint_{\mathbf{k}\in \mathrm{C}(0,|\mathbf{k_0}|,k_{res})} |W_{\mathbf{k_0}}\Psi_{\mathbf{k_0}}(\mathbf{k})|^2 \mathrm{d}^3\mathbf{k}} \tag{13}$$

where $\mathrm{S}(\mathbf{k_0},k_{res})$ is the sphere centered on $\mathbf{k_0}$ with radius k_{res}, and where $\mathrm{C}(0,|\mathbf{k_0}|,k_{res})$ is either the volume delimited by the two spheres with center O and of radii $|\mathbf{k_0}| - k_{res}$ and $|\mathbf{k_0}| + k_{res}$ if $|\mathbf{k_0}| > k_{res}$, or only the sphere of radius $|\mathbf{k_0}| + k_{res}$ otherwise. Thus, the numerator represents the energy of the spatially sampled Fourier atom $\psi_{\mathbf{k_0}}$ contained in the sphere of radius k_{res} surrounding the wave vector of interest $\mathbf{k_0}$, whereas the denominator is the total power in a spherical crown of radius $|\mathbf{k_0}|$. Granted that at pulsation ω_0, the spectrum $P(\mathbf{k},\omega_0)$ is supposed to be excited inside this crown, the weights $w_n(\mathbf{k_0})$ which maximize λ minimize the interference between the truncated Fourier atom and the spectrum of the sound field being analyzed. The criterion (13) is inspired by spectral analysis based on prolate spheroidal wave functions [11].

The Fourier transform of the truncated Fourier atom $W_{\mathbf{k_0}}\Psi_{\mathbf{k_0}}$ is given by the following relation:

$$\begin{aligned}W_{\mathbf{k_0}}\Psi_{\mathbf{k_0}}(\mathbf{k}) &= \sum_{n=1}^{N} \mathscr{V}\,\overline{w}_n e^{\mathrm{i}(\mathbf{k_0}-\mathbf{k})\cdot\mathbf{r_n}} \\ &= \underline{w}^H \underline{s}\end{aligned} \tag{14}$$

$$\text{with } \underline{w} = [w_1,\ldots,w_N]^T$$

$$\text{and } \underline{s} = \mathscr{V}\left[e^{\mathrm{i}(\mathbf{k_0}-\mathbf{k})\cdot\mathbf{r_1}},\ldots,e^{\mathrm{i}(\mathbf{k_0}-\mathbf{k})\cdot\mathbf{r_N}}\right]^T$$

where subscript ^{H} denotes conjugate transposition. From the above equation (14), we deduce the following matrix formulation for equation (13):

$$\begin{aligned}\lambda &= \frac{\underline{w}^H\left[\iiint_{\mathbf{k}\in \mathrm{S}(\mathbf{k_0},k_{res})}\underline{s}\,\underline{s}^H \mathrm{d}^3\mathbf{k}\right]\underline{w}}{\underline{w}^H\left[\iiint_{\mathbf{k}\in \mathrm{C}(0,|\mathbf{k_0}|,k_{res})}\underline{s}\,\underline{s}^H \mathrm{d}^3\mathbf{k}\right]\underline{w}} \qquad (15)\\ &= \frac{\underline{w}^H \mathscr{S}_{res}\underline{w}}{\underline{w}^H \mathscr{S}_{tot}\underline{w}}\end{aligned}$$

$$\text{with } \mathscr{S}_{res}(m,n) = \iiint_{\mathbf{k}\in \mathrm{S}(\mathbf{k_0},k_{res})} \mathscr{V}^2 e^{\mathrm{i}(\mathbf{k_0}-\mathbf{k})\cdot(\mathbf{r}_m-\mathbf{r}_n)}\mathrm{d}^3\mathbf{k}$$

$$\text{with } \mathscr{S}_{tot}(m,n) = \iiint_{\mathbf{k}\in \mathrm{C}(0,|\mathbf{k_0}|,k_{res})} \mathscr{V}^2 e^{\mathrm{i}(\mathbf{k_0}-\mathbf{k})\cdot(\mathbf{r}_m-\mathbf{r}_n)}\mathrm{d}^3\mathbf{k}$$

The above problem is equivalent to resolve the generalized eigenvalue problem [11]:

$$\mathscr{S}_{res}\underline{w}_i = \lambda_i \mathscr{S}_{tot}\underline{w}_i \qquad (16)$$

There are N eigenvalues of this problem, and the maximal one is the solution of the optimization criterion (13).

4. PERFORMANCE OF THE ESTIMATION

In this part, several configurations of microphone arrays are investigated, such as bi-dimensional and tri-dimensional microphone arrays. The efficiency of the criterion introduced in part 3.2 is shown, and the performance are compared to uniform analysis windows. Moreover, the performance are compared on the global bandwidth of sound signals, that is between the range $20\mathrm{Hz}-20\mathrm{kHz}$.

4.1. Influence of the array geometry

Two kinds of array are compared in this paragraph: the first one is a bi-dimensional array constituted of 5 circular arrays having 10 elements each for several radii logarithmically-spaced between $0.01-1\mathrm{m}$, plus an additional sensor at the origin [10]; the second one is a tri-dimensional array constituted of several Platonic solids —in this order, an octahedron, a dodecahedron, an isocahedron, a cube and a tetrahedron, which are the only regular meshes of the sphere— inscribed in a sphere of a given radius, for several radii logarithmically-spaced between $0.01-1\mathrm{m}$, plus an additional sensor at the origin. Both arrays have 51 elements. The interest of using several subarrays with different radii is that each one is adapted for a given frequency band. Indeed, it seems relevant that an array with radius R is a good choice to analyze wavelengths of the same order of magnitude, and implicitly, with the dispersion relationship, is dedicated to a particular frequency band. So the use of multiple radii is to improve the performance of the global array along the whole frequency bandwidth of sound fields.

Figure 1 represents the performance —evaluated by the criterion introduced at equation (13)— achieved by the two array configurations for uniform and optimal analysis windows, using the following parameters $\mathbf{k_0}(\omega_0) = [\omega_0/c, 27\pi/180, 27\pi/180]$ in spherical coordinates, for several values of f_0 spanning the entire bandwidth $20\mathrm{Hz}-20\mathrm{kHz}$. It is seen that optimal analysis windows computed by (16) outperform uniform analysis windows for the two arrays. Moreover, the use of a tri-dimensional array gives better results than the bi-dimensional array up to 10kHz: tri-dimensional arrays have better discrimation capabilities than bi-dimensional arrays. The crossing of the two curves around 10kHz is linked to the fact that 10 microphones are used in the subarray of radius $0.01\mathrm{m}$ in the bi-dimensional case compared to the 6 vertex of the corresponding octahedron in the tri-dimensional array.

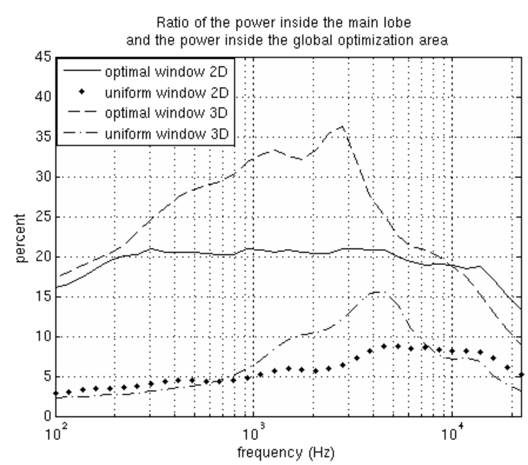

Fig. 1: Criterion (13) evaluated for two array configurations, bi-dimensional and tri-dimensional, for optimal and uniform analysis windows, versus frequency, with $\mathbf{k_0}(\omega_0) = [\omega_0/c, 27\pi/180, 27\pi/180]$

4.2. Comparison between several estimation methods

In this paragraph, we are interested by the representation of the modulus of the Fourier transform of a sampled plane wave, with the following parameters: $f_0 = 2039\text{Hz}$ and $\mathbf{k_0} = [\omega_0/c, 27\pi/180, 27\pi/180]$ in spherical coordinates. The first representation plots $\psi^{sam}_{\mathbf{k_0},\omega_0}(k_0,\phi_k,\theta_k,\omega_0)$, in a similar manner than a directivity diagram extended to three dimensions. The second representation plots the modulus of the Fourier transform, mapping the sphere on a rectangular grid, in decibels.

The Fourier transforms of the sampled plane wave using uniform analysis windows are represented on figures 2 and 4 for the bi-dimensional and tri-dimensional array respectively. And the Fourier transforms of the sampled plane wave using optimal analysis windows are represented on figures 3 and 5.

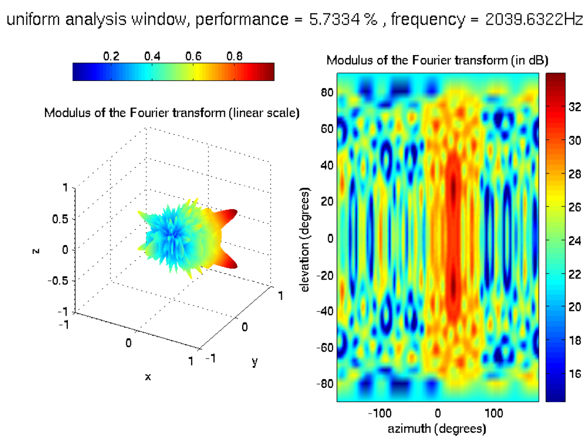

Fig. 2: Fourier transform of the sampled plane wave with uniform weighting for bi-dimensional array

It is seen that better resolution is obtained using uniform analysis windows. On the other hand, the sampled plane wave is very badly localized in the wave vector domain using uniform analysis windows compared to optimal analysis windows, because there are many high-level sidelobes. Optimal analysis windows achieve the best tradeoff between resolution and rejection of the sidelobes.

An interesting observation is that the Fourier transform of the sampled plane wave using the bi-dimensional array (figures 2 and 3) is symmetric with respect to the plane (Oxy), confirming the fact that no distinction between up and down is possible using this kind of array.

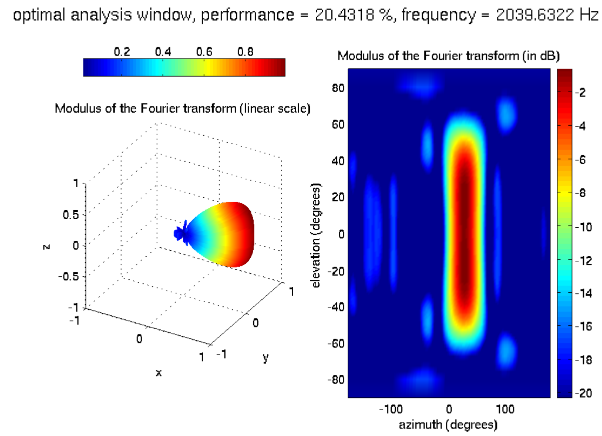

Fig. 3: Fourier transform of the sampled plane wave with optimal weighting for bi-dimensional array

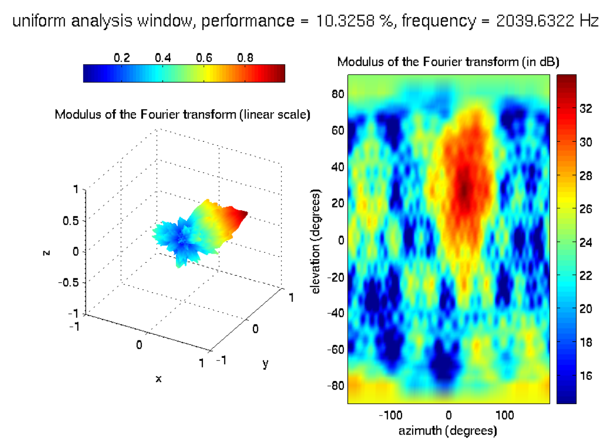

Fig. 4: Fourier transform of the sampled plane wave with uniform weighting for tri-dimensional array

The best performance of focalization of the power inside the main lobe, 33%, is obtained at this frequency using the optimal analysis window with the tri-dimensional array described at section 4.1. This configuration minimizes the spatial aliasing between the wave vector of interest $\mathbf{k_0}$ and the sound field being analyzed so that its integration as a part of a spatial analysis processing module is very well indicated.

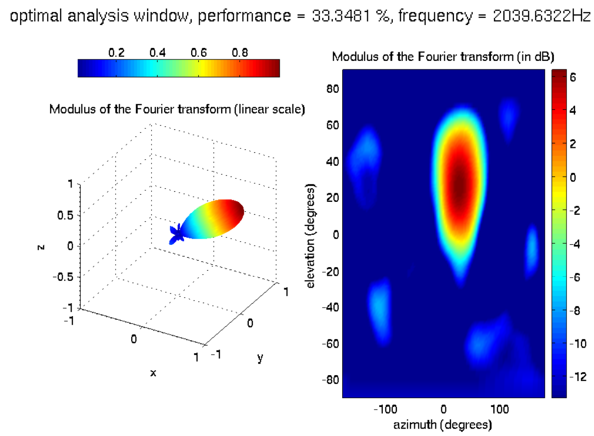

Fig. 5: Fourier transform of the sampled plane wave with optimal weighting for tri-dimensional array

5. CONCLUSION

In this paper, it has been shown that a microphone array introduces inevitably spatial aliasing. It has been linked to the Fourier transform of the analysis window, which is composed of a mainlobe and also from sidelobes, which are very undesirable for spatial analysis purpose. This Fourier transform depends on the geometry of the array, and also from the weights applied to each element of the microphone array. Granted that the sampled sound field is localized in the space domain due to the microphone array, it is no longer localized in the wave vector domain. So, the method of design of the optimal analysis window has been made in order to achieve the best localization possible in the wave vector domain, with respect to a particular wave vector. The efficiency of the approach has been demonstrated compared to uniform analysis windows. The sound field analysis presented in this paper enables also the analysis of high frequencies, contrary to traditional analysis methods.

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