ON THE DIRECTIONAL LOCALISATION OF SOUND IN THE STEREOPHONIC SOUND FIELD

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Summary,

The phenomenon of directional localisation of the sound in stereophony may be explained by assuming that the apparent direction of a source is that of the normal to the wave-front. That direction may be calculated, for the general case of a double-channel stereophonic system, in terms of the conditions in the studio and in the listening room. Some anomalies of the stereophonic wave-front, compared with direct listening to a real source, are then discussed, and their influence on directional localisation are considered. This discussion deals essentially with, on the one hand, the differences between the velocity of propagation of the wave-front and the velocity of sound and, on the other hand, the variation of the direction perpendicular to the wave-front as a function of frequency in the case where two spaced microphones are used. Finally, the conclusions of the theory are compared with the results of experimental determinations in the direction of the sound image in stereophony.

INTRODUCTION

In an ordinary sound field, all the information about the direction of the sound source is borne by the direction of propagation of the direct sound waves from the source, the direction of propagation of the direct sound waves being that of the normal to the wave-fronts.

The differences that occur at the two ears of a listener, that is to say intensity-difference and arrival-time or phase-difference, correspond to the direction of the normal to the wave-front of the direct sound from the sound source. In other words, they act as clues that enable the listener to perceive the direction of the normal to the wave-front and thus the direction of the source of sound.

The differences in intensity and phase at the two ears as functions of the direction of sound have been thoroughly investigated by many authors. The theories so far presented [1] – [4] on the problem of the directional localisation of sound perceived by the listener in the two-channel stereophonic sound field are founded on these differences. Nevertheless, they do not entirely explain the phenomena of the directional localisation of stereophonic sound

In the present paper, another method of attacking the problem is described. In this method, the direction of propagation of the wave-front in the two-channel stereophonic sound field was investigated as a function of the studio and listening conditions, assuming that the listener perceives the image of reproduced sound in that direction, as is the case with everyday experiences. In addition, some anomalous characteristics of the wave-fronts of stereophonic sound fields are made clear, together with their effects on the directional localisation.

THE CHARACTERISTICS OF THE WAVE-FRONT IN THE STEREOPHONIC SOUND FIELD

The velocity potential in a two-channel stereophonic sound field has been deduced in terms of the source and listening conditions. The characteristics of a surface with a given phase, namely that of the wave-front, was investigated as a function of the direction of the normal to the wave-front, the velocity of propagation of the wave-front, and the variation of the velocity potential along the wave-front.

Some of these characteristics are anomalous compared with those of the wave-fronts encountered in everyday experiences.

The Direction of the Normal to the Wave-Front in a Stereophonic Sound Field.

Fig. 1(a) shows the arrangement of two microphones (M_1 and M_2) and a point source of sound (S) in the source-space.

The distance between the two microphones is $2d_m$. Let the distance from S to M_1, M_2 and O', the middle point of the line joining the two microphones, be denoted by r_1, r_2, and r_o respectively. The direction of the sound source is designated by θ measured counter-clockwise from the positive sense of the x′-axis constituted by the line $M_1\ O'\ M_2$.

Fig. 1(b) shows the arrangement of two loudspeakers (L_1 and L_2) and a listener (H) in the listener-space.

Let the position of the middle point between the two loudspeakers be the origin O of the coordinates and the line $L_1\ O\ L_2$ be the x-axis in the listener-space. The coordinates of the loudspeakers L_1, L_2 and listener H are designated by $(-d_L, 0)$, $(d_L, 0)$, and $(x, 0)$ respectively. Let the distance from the listener to L_1, L_2, and O be denoted by R_1, R_2 and R_o respectively.

Let signals proportional to the output voltages of the microphones M_1 and M_2 in the sound field of the point source of sound S, of which the intensity is $4\pi A$ and the frequency is f, be applied independently to the corresponding loudspeakers L_1 and L_2. Then the direction of the normal towards the interior of the surface to the wave-front of the resultant sound field at the position of the listener H can be calculated as follows:·

In the source-space, the velocity potentials Φ_{M1} and Φ_{M2} at M_1 and M_2 are given by

$$\Phi_{M1} = \frac{A}{r_1} \exp jk(ct - r_1)$$

$$\Phi_{M2} = \frac{A}{r_2} \exp jk(ct - r_2) \qquad (1)$$

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*Mr. Makita is with the Nippon Hoso Kyokai. This article is based upon an article read by him at the 12th Meeting of the Technical Committee of the E.B.U., Monte Carlo, October, 1960.

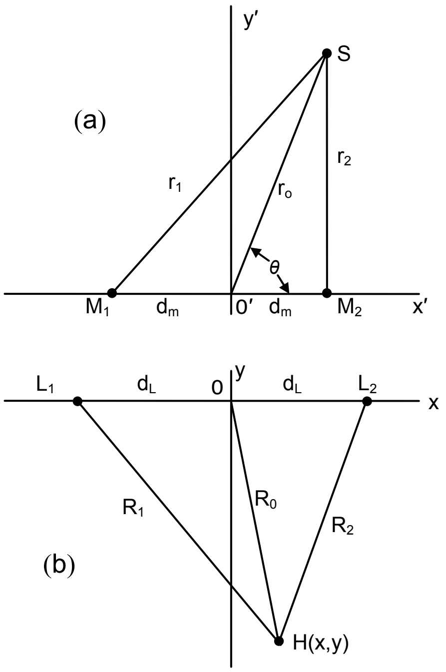

Fig. 1 --- Geometry of

(a) microphones M1 and M2 and the source S in the loudspeaker space, and

(b) loudspeakers L1 and L2 and the listener H in the listener-space

where $k = \frac{2\pi f}{c}$

and c = the velocity of propagation of sound.

Since d_m is usually smaller than r_o, the following first approximation may be allowed:-

$$r_1 = r_o\left(1 + \frac{d_m}{r_o}\cos\theta\right)$$

$$r_2 = r_o\left(1 - \frac{d_m}{r_o}\cos\theta\right) \tag{2}$$

Substituting Eq. (2) in Eq. (1) yields

$$\Phi_{M1} = \frac{A}{r_o}\left(1 - \frac{d_m}{r_o}\cos\theta\right)\exp jk\left[ct - r_o(1 + \frac{d_m}{r_o}\cos\theta)\right]$$

$$\Phi_{M2} = \frac{A}{r_o}\left(1 + \frac{d_m}{r_o}\cos\theta\right)\exp jk\left[ct - r_o(1 - \frac{d_m}{r_o}\cos\theta)\right] \tag{3}$$

Hence, provided that $(1/kr_o)^2$ is negligibly small compared with unity, the output signals of the two microphones can be expressed as follows, independently of the kind of microphone (pressure microphone, pressure-gradient microphone, or combined microphone, etc.) :-

$$E_{m1} = \frac{KA}{r_o}\left(1 - \frac{d_m}{r_o}\cos\theta\right)\exp jk\left[ct - r_o(1 + \frac{d_m}{r_o}\cos\theta)\right]$$

$$E_{m2} = \frac{\gamma KA}{r_o}\left(1 + \frac{d_m}{r_o}\cos\theta\right)\exp jk\left[ct - r_o(1 - \frac{d_m}{r_o}\cos\theta)\right] \tag{4}$$

where γ = ratio of the sensitivity of microphone M_2 to that of microphone M_1, and

$$K = k\rho c\xi$$

where ρ = density of air
and ξ = the sensitivity of microphone M_1

When directional microphones are used, γ is a function of θ and η, the direction of the axis of directivity of the microphones. In the case of the uni-directional (cardioid) microphones, γ is obtained as follows:-

$$\gamma = \frac{1 + \cos(\theta - \eta)}{1 - \cos(\theta + \eta)} \tag{5}$$

Any differences between the two channels in amplification and in loudspeaker response may, of course, also be included in γ.

Let the volume velocities of the two loudspeakers driven by the signals proportional to E_{m1} and E_{m2} be $4\pi A_{L1}$ and $4\pi A_{L2}$ respectively. For the sake of simplicity, we shall consider the loudspeakers as point sources.

Thus, the velocity potential Φ_H of the resultant sound field at the position (H) of a listener can then be written as follows:-

$$\Phi_H = \frac{A_{L1}}{R_1}\exp\left[-j(kR_1 - \frac{\varepsilon}{2})\right] + \frac{A_{L2}}{R_2}\exp\left[-j(kR_2 - \frac{\varepsilon}{2})\right] \tag{6}$$

where

$AL_1 = \mu E_{m1}$ $\qquad$ $AL_2 = \mu E_{m2}$

μ = the system electro-acoustic conversion factor and
r = phase difference between the channels.

Eq. (6) can be reduced to the following form,

$$\Phi_H = \frac{K'}{r_0}\sqrt{X^2 + Y^2}\,\exp\left[jk(ct - r_o - \frac{R_1 + R_2}{2} - \frac{\chi}{k})\right] \tag{7}$$

where

$$K' = AK_\mu$$

$$X = \left(\frac{\alpha}{R_1} + \frac{\gamma\beta}{R_2}\right)\cos k\varphi$$

$$Y = \left(\frac{\alpha}{R_1} - \frac{\gamma\beta}{R_2}\right)\sin k\varphi$$

$$\chi = \tan^{-1}\frac{Y}{X}$$

$$\varphi = td_m\cos\theta + \frac{R_1 - R_2}{2} + \frac{\varepsilon}{2k}$$

$$\alpha = 1 - \frac{d_m}{r_o}\cos\theta$$

$$\beta = 1 + \frac{d_m}{r_o}\cos\theta$$

$$R_1 = \sqrt{(x + d_L)^2 + \gamma^2}$$

$$R_2 = \sqrt{(x - d_L)^2 + \gamma^2} \qquad (8)$$

From Eq. (7), the following equation of the curve may be deduced, from which the wave-front for a given phase angle δ, meets the horizontal plane L1HL2.

$$\Phi(x, y) = k(ct - r_o - \frac{R_1 + R_2}{2} - \frac{\chi}{k}) = \delta \qquad (9)$$

Let the direction of the normal to the wave-front at (x,y) be denoted by Θ, which is measured counter-clockwise from the positive sense of the x-axis. From Eq. (9), Θ can be obtained as follows:

where *s* is a small displacement along the wave-front.

(a) When *H* is on the y-axis, Eq. 20 becomes

$$\frac{\partial \Phi_H}{\partial s} = \frac{K'}{r_o\sqrt{\alpha^2+\gamma^2+\beta^2+2\alpha\gamma\beta(\cos^2 k\varphi - \sin^2 k\varphi)}} - \frac{d_L}{R^2}(\alpha+\gamma\beta)\left\{\frac{\alpha+\gamma\beta}{R} + k(\alpha+\gamma\beta)\sin k_\varphi \cos k_\varphi\right\}\sin\theta$$

$$-\frac{y}{R^3}\left\{\alpha^2+\gamma^2\beta^2+2\alpha\gamma\beta(\cos^2 k\varphi - \sin^2 k\varphi)\right\}\cos\theta \exp jk(ct - r_o - \frac{R_1+R_2}{2} - \frac{\chi}{k}) \qquad (21)$$

Eq. (21) shows that magnitudes of the variation of velocity potential $\frac{\partial \Phi_H}{\partial s}$ along the wave-front decreases as H moves back along the y-axis from the centre of the two loudspeakers. In general, this variation is negligibly small.

(a) In the case when $d_m = 0$ (coincident microphones being used) Eq. (19) becomes

$$\frac{\partial \Phi_H}{\partial s} = 0$$

as is often the case with everyday experience.

The foregoing formulae therefore make it possible to determine those characteristics of the wave-front that are involved in the phenomenon of the directional localisation of sound in stereophony.

TABLE 1. —- Values of n = c/c' calculated by Eq. (19)

γ (dB)	$\frac{y}{d_L} = 2.414$	$\frac{y}{d_L} = \sqrt{3}$	$\frac{y}{d_L} = \frac{1}{\sqrt{3}}$
0	0.9239	0.8660	0.5000
0.2	0.9239	0.8660	0.5001
0.4	0.9239	0.8661	0.5004
0.6	0.9240	0.8662	0.5009
0.8	0.9240	0.8663	0.5016
1	0.9241	0.8665	0.5025
2	0.9249	0.8679	0.5098
4	0.9279	0.8733	0.5370
6	0.9326	0.8818	0.5769
8	0.9416	0.8924	0.6237
10	0.9450	0.9041	0.6726
14	0.9585	0.9281	0.7642
17	0.9677	0.9442	0.8114
20	0.9755	0.9578	0.8672
25	0.9708	0.9670	0.8973
30	0.9718	0.9686	0.9027

DIRECTIONAL LOCALISATION OF SOUND IN THE TWO-CHANNEL STEREOPHONIC SOUND FIELD

The direction of the sound image perceived by a listener in the two-channel stereophonic sound field may be defined by *θ*, given by Eqs. (10) to (14) for various studio and listening conditions, provided that the nature of the wave-fronts discussed in those sections is not too different from that of the wave-fronts encountered in everyday experience.

Case when coincident microphones are used…

If coincident microphones are used, anomalies of the wave-front almost disappear under the following listening conditions; (a) symmetrical listening and (b) the listener keeps further from the centre of the two loudspeakers than $\sqrt{3}/d_L$. In this case, therefore, the directional localisation of the sound must be defined by *θ* as given by Eq. (14).

In order to check the validity of this reasoning, the calculated values of θ from Eq. (14) are compared in *Table II* with the experimental results obtained by H. Mertens in a preliminary series of experiments undertaken in an anechoic room at the University of Liège.*

The experiments were carried out under the following conditions which were also applied in the calculation:-

(i) The signals applied to the two loudspeakers were Gaussian pulses without either time or phase difference but with a voltage difference,

(ii) The loudspeakers were spaced three metres apart and formed with the observer an equilateral triangle.

The experimental result shown in Table II, are the average of data obtained by ten observers. They are in fair agreement wit the values obtained theoretically from Eq. (14).

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* *As part of the programme of work on this problem being undertaken by the E.B.U. Technical Centre, further experiments have since been effected at the University of Liège and elsewhere. Although they have in general been in fairly satisfactory agreement with the earlier results cited above by the author of the present paper (when the dispersion resulting from the subjective nature of the method is taken into consideration, the most recent results correspond to slightly, but systematically, smaller values of the angle* θ. *We intend to publish in the near future a detailed description of these experiments, together with a discussion of the results. (Editor)*

TABLE II. Experimental values of the direction

γ (dB)	Values observed at the University of Liege for the following frequencies:					Calculated values		
	310 c/s	500 c/s	1100 c/s	2200 c/s	4200 c/s	Θ	n	Θ′
0	90.0	90.0	90.0	90.0	90.0	90.0	0.8660	90.0
2	84.9	85.6	86.0	85.6	87.4	86.2	0.8679	86.7
4	80.2	81.5	82.4	81.3	84.6	82.6	0.8732	83.5
6	76.6	78.2	79.2	77.0	81.3	79.2	0.8818	80.5
8	73.3	75.3	76.0	73.7	77.4	76.1	0.8924	77.6
10	70.8	73.0	73.4	70.8	73.4	73.3	0.9041	74.9
12	68.9	71.5	70.9	69.0	70.6	70.8	0.9162	72.5
14	67.6	69.4	68.7	67.7	68.4	69.0	0.9281	70.6
16	66.5	67.6	66.9	66.6	66.6	67.2	0.9391	68.7
18	65.4	66.2	65.4	65.8	65.2	65.7	0.9490	67.0
20	64.6	64.8	64.3	64.8	64.0	64.6	0.9578	65.7
25	63.1	62.2	62.2	62.8	61.9	62.7	0.7670	63.7
30	61.8	61.0	60.9	61.6	60.8	61.6	0.9686	62.6
40	60.2	60.0	60.0	60.2	60.0	60.5	0.9251	60.7

Effect of variations of the velocity of propagation of the wave-front on the directional localisation.

The direction of the sound image θ' perceived by a listener facing the loudspeakers when the velocity of propagation c' of the wave-front differs from the velocity of sound c, may be derived from θ by the simple assumption that θ' is the normal to the wave-front having the velocity of propagation c, that produces the same time-difference at the listener's ears as the wave-front with the velocity of displacement c' produces.*

According to this reasoning, the following relation between θ' and θ may be derived:-

$$\cos\theta' = \frac{c}{c'}\cos\theta = n\cos\theta \quad (22)$$

The corrected values of θ' are shown in the last column in Table II. It will be noted that it is not possible to say whether θ or θ' shows better overall agreement with the experimental results.

In order to make clear the effect of the velocity of propagation of the wave-front, further experiments were carried out under the following conditions:-

(1) The signals applied to the two loudspeakers were "white noise", without either time or phase difference, but with a voltage difference.

(2) The loudspeakers were spaced 2.7 metres apart. Observations were made with the listener symmetrically disposed and at three values of the distance y from the centre of the line joining the two loudspeakers.

The averages of the data obtained by six observers are shown in *Table III*, together with, for comparison, the calculated values of $n = c/c'$, Θ and Θ'.

The effect of the velocity of propagation of the wave disappears when the listener turns his face to the wave-front. Furthermore, the variation [1] of the direction of the sound image perceived by the listener in the stereophonic sound field, when he turns his head, may be explained by the change of the effect of the velocity of propagation of the wave-front on θ', as a function of the angular difference between θ and the direction of his head

Influence of frequency of sound source on the direction of the normal to the wave-front in the stereophonic sound field.

The angle θ given by Eq. (10) to Eq. (12) is a function of $k = 2\pi f/c$, that is to say of the frequency of the sound. This means that the normal to the wave-front at a given point in the sound field in stereophonic reproduction may vary with the frequency of the source, although it remains at a fixed position. Moreover, when the sound source radiates a complex sound,

TABLE III. ---- Influence of the velocity of the wave-front. Experimental and theoretical values, uncorrected (θ) and corrected (θ') respectively, of the direction of the sound image (in degrees).

y	y (dB)	Values observed	Calculated values		
			Θ	n	Θ'
2.34 m	5.5	79.5	80.1	0.8793	81.3
	13.8	67.5	69.1	0.9271	70.7
1.35 m	4.8	75.0	75.0	0.7320	79.1
	8.1	67.5	66.5	0.7710	72.1
0.78 m	4.3	75.0	67.4	0.5411	78.0
	7.2	67.5	56.0	0.6033	70.3

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* a method of correction using the variation of intensity differences could equally well be used.

Fig. 2 --- Effect of the source frequency on the apparent direction of the sound image, for two omnidirectional microphones spaced at 60 cm. The angles plotted as ordinates are measured from the axis constituted by the straight line joining the two loudspeakers.

Each component of the reproduced sound has a wave-front with its own normal direction. This is not the case with the wave-front of a direct sound radiated from a real source.

For example, computing Eq. (12) numerically for the following case, let us determine how θ varies with the frequency of the sound source,

$\theta = 60^\circ$ $r_o = 3{,}0$ m $d_m = 30$ cm $\gamma = 1$

$\varepsilon = 0$ $2d_L = 3$ m $x = 0$ $y = -2{,}5$ m

Fig. 2 shows the results of this calculation. We find a remarkable variation of θ around the frequencies 566 c/s, 1698 c/s … $(2n + 1)$ X 566 c/s, which satisfy the following equation

(23)

where $n = 0, 1, 2, \dots$

However, in the frequency range well below the lowest anomalous frequency given by Eq. (23), θ is independent of the frequency of the sound source.

The anomalous frequencies become higher as the spacing between the two microphones decreases or the direction of the sound source approaches the centre.

In order to check the validity of the theory, the following experiments were carried out under the above conditions.

In an anechoic chamber (7,7 X 7,8 X 7,3 m^3) a loudspeaker (30 cm in diameter) radiated a sinusoidal tone of which the frequency was varied from 100 c/s to 3200 c/s. The tone was picked up by two non-directional microphones spaced 60 cm apart.

In another anechoic chamber (5,6 X 5,2 X 4,4 m^3), two loudspeakers (30 cm in diameter) were spaced 3 metres apart and an observer (one of two observers) was situated at an equal distance of 2.9 metres from each loudspeaker. The ears of the observer and the two loudspeakers were in the same horizontal plane. The overall frequency characteristics of the two channels were equal within ± 2 dB.

The observers were in turn requested to point to the apparent source of sound, the direction of which was observed on a graduated screen hung between the observer and the loudspeakers. The average of the localised directions of the apparent source of sound was plotted in Fig. 2. While the results obtained do not show exact agreement with the calculated values, it confirmed that the anomalous frequencies exist, as the theory predicted.

CONCLUSION

The characteristics of the wave-front in the two-channel stereophonic sound filed has been investigated theoretically, as functions of the studio and listening conditions, assuming that the listener localises the sound in the direction of propagation of the wave-front.

Certain anomalies of the wave-front in the stereophonic sound field have come to light, but their effects on the localisation have not yet been completely determined.

Nevertheless, the results so far obtained may well make some contribution to the solution of certain problems in stereophony.

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ACKNOWLEDGEMENTS

The author wishes to express his thanks to Mr. S. Shima, formerly Director of the Technical Research Laboratories of the N.H.K., for his encouragement throughout this work as well as to Mr. G. Hansen and Mr. H. Mertens of the E.B.U. Technical Centre for their valuable suggestions and discussions.

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